J. N. KAPUR

THE SPIRIT OF MATHEMATICS

ARYA BOOK DEPOT
NEW DELHI-5

2602
30-3-77

THE SPIRIT OF MATHEMATICS

(A collection of sayings of Great Mathematicians and Scientists is all Ages about the Nature and Applications of Mathematics.)

PROF. J. N. KAPUR
Head of Mathematics Deptt.,
I.I.T., Kanpur.

Published by

ARYA BOOK DEPOT
30, NAI WALA, KAROL BAGH,
NEW DELHI—5.

Published by :
Arya Book Depot,
Karol Bagh, New Delhi-5.

Price : Rs. 5·00

Printed at :
Sikksons' Press,
Karol Bagh, New Delhi-5.

PREFACE

The problem of the nature of mathematics has greatly fascinated the mathematicians and scientists of all ages. Apart from the fascination, the subject is of great practical interest, in as much as ideas about the nature of mathematics are greatly influencing the nature of both teaching and research effort in mathematics. As such it is necessary that these thoughts should be available to the students, teachers and research workers in mathematics and to all those workers in the fields of Physical, social and biological sciences and in industry and technology who have become interested in the nature of mathematics due to some of the important advances that have been made possible in their fields with the help of mathematics.

The present collection gives the answers to the question 'What is Mathematics ?' given by the great thinkers of all times.

In order to derive the maximum benefit, one must not only read each thought, but must think it over and must illustrate it from one's own experience.

It is hoped that the present collection will be useful to all interested in Mathematics and will help in inspiring students, teachers and research workers in mathematics to greater efforts.

I.I.T.,
KANPUR

J. N. Kapur

THOUGHTS ABOUT THE NATURE OF MATHEMATICS

1. Mathematics is the science of number and space.
—DICTIONARY

2. Mathematics is the science of measurement, quantity and magnitude.
—DICTIONARY

3. Mathematics in its widest significance is the development of all types of formal deductive reasoning.
—A.N. WHITEHEAD

4. Pure mathematics is the class of all propositions of the form *p* implies *q*, where *p* and *q* are propositions containing one or more variables, the same in two propositions and neither *p* nor *q* containing any constants except logical constants. —BERTRAND RUSSEL

5. Mathematics is the science which draws necessary conclusions. —PIERCE

6. Mathematics, in general, is fundamentally the science of self-evident things. —FELIX KLEIN

7. Mathematics is nothing more than a game played according to certain rules with meaningless marks on paper. —DAVID HILBERT

8. Mathematicians do not study objects, but the relations between them. Matter does not engage their attention, they are interested in form alone. —H. POINCARE

9. A mathematical system is any set of strings of recognizable marks in which some of the strings are taken initially and the remainder derived from these by operations performed according to rules which are independent of any meanings assigned to the marks. —C I. LEWIS

10. Mathematics includes computation with natural numbers and everything that can be founded on it, but nothing else. —STUDY

11. The science of pure mathematics, in its modern development, may claim to be the most original of the human spirit. —WHITEHEAD

12. A mathematician who is not also something of a poet will never be a complete mathematician. —WEIERSTRASS

13. I have heard myself accused of being an opponent, an enemy of mathematics, which no one can value more highly than I, for it accomplishes the very things whose achievement has been denied me. —GOETHE

14. Mathematicians are like lovers. Grant a mathematician the least principle and he will draw from it a consequence which you must also grant him, and from this consequence another. —FONTENELLE

15. I regret that it has been necessary for me in this lecture to administer such a large dose of four-dimensional geometry. I do not apologize, because I am really not responsible for the fact that nature in its most fundamental aspects is four-dimensional. Things are what they are. —WHITEHEAD

16. Numbers rule the universe. —The Pythagoreans

17. Mathematics is the queen of the sciences, and arithmetic the queen of mathematics. —Gauss

18. Thus numbers may be said to rule the whole world of quantity, and the four rules of arithmetic may be regarded as the complete equipment of the mathematician. —Maxwell

19. God made the integers, all the rest is the work of man. —Kronecker

20. Arithmetic is one of the oldest branches, perhaps the very oldest branch of the human knowledge, and yet some of its most obstruse secrets lie close to its tritest truth. —Smith

21. Let no one ignorant of geometry enter here. —Plato

22. There is no royal road to geometry. —Menaechmus

23. Strange as it may sound, the power of mathematics rests on its evasion of all unnecessary thought and its wonderful saving of mental operation. —Ernst Mach

24. A single curve, drawn in the manner of the curve of prices of cotton, describes all that the ear can possibly hear as the results of the most complicated musical performance ; that to my mind is a wonderful proof of the potency of mathematics. —Lord Kelvin

25. He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and language. Hence his fondness for Euclid, which he carried with him on the circuit till he could demonstrate with ease all the propositions in the six books. —Said About Lincoln

26. The mathematician, carried along on his flood of symbols, dealing apparently with pure formal truths,

may still reach results of endless importance for our description of the physical universe.

—Karl Pearson

27. Examples which might be multiplied alibitum, show how difficult it often is for an experimenter to interpret his results without the aid of mathematics.

—Lord Rayleigh

28. But there is another reason for the high repute of mathematics; it is mathematics that affords the exact natural sciences, a certain measure of security which, without mathematics, they could not attain.

—Albert Einstein

29. Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field. For this reason a book on the new physics, if not purely descriptive of experimental work, must be essentially mathematical.

—P.A.M. Dirac

30. As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena (of electromagnetism) was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms and thus compared with those of the professional mathematicians.

—Maxwell

31. To create a healthy philosophy, you must renounce metaphysics, but be a good mathematician.

—Bertrand Russel

32. Mathematics is the only good metaphysics.

—Lord Kelvin

33. How can it be that mathematics, being after all a product of human thought, independent of experience, is so admirably adapted to the objects of reality ?

—Albert Einstein

34. Every new body of discovery is mathematical in form, because there is no other guidance we can have.
—G. C. Darwin

35. The infinite ? No other question has ever moved so profoundly the spirit of man. —David Hilbert

36. The notion of infinity is our greatest friend, it is also the greatest enemy of our peace of mind. Weierstrass brought us to believe that we had at least thoroughly tamed and domesticated this unruly element. Such however is not the case, it has broken loose again. Hilbert and Brouwer have set out to tame it once more. For how long ? We wonder.
—James Pierpont

37. In my opinion, a mathematician, in so far he is a mathematician, need not preoccupy himself with philosophy—an opinion, moreover which has been expressed by many philosophers. —Henri Lebesgue

38. God ever geometrises. —Plato

39. God ever arithmetises. —C. G. Jacobi

40. The Great Architect of the universe now begins to appear as a pure mathematician. —J. H. Jeans

41. Mathematics is the most exact science, and its conclusions are capable of exact proof. But this is so only because mathematics does not attempt to draw absolute conclusions. All mathematical truths are relative, conditional. —C. F. Steinietz

42. It is a safe rule to apply that, when a mathematical or philosophical author speaks with a misty profoundity, he is talking nonsense. —A N. Whitehead

43. There is probably no other science which presents such different appearances to one who cultivates it and one who does not, as mathematics. To the non-cultivators, it is ancient, venerable and complete ; a body of dry irrefutable unambiguous reasoning. To the mathe-

matician, on the other hand, his science is yet in the purple bloom of vigorous youth, everywhere stretching out after the "attainable but unattained", and full of the excitement of nascent thought, its logic is beset with ambiguities and its analytic processes like Bunyans Road have a quagmire on one side and a deep ditch on the other, and branch off into innumerable by-paths that end in wilderness. —CHAPMAN

44. Mathematics may be regarded either as a body of achievements or as an intellectual enterprize.

—KEYSER

45. It was in mathematics that deductive reasoning first appeared and it is from the same source that its successive extensions and refinements have issued. In its most powerful form, deductive reasoning is mathematical. The logical apparatus used in mathematics is incomparably more varied, more subtle, and more creative of new combinations than that associated with any other field of knowledge. —KEYSER

46. The mathematical method is by far the most effective and most powerful method yet devised for the description of the various aspects of nature.

47. In every case the awakening touch has been the mathematical spirit, the attempt to count, to measure, or to calculate. What to the poet or the seer may appear to be the very death of all his poetry and all his visions, the cold touch of the calculating mind, this has proved to be the spell by which knowledge has been born, by which new sciences have been created and hundreds of definite problems put before the minds and hands of diligent students. It is the geometrical figure, the dry algebraical formulae which transform the vague reasoning of the philosopher into a tangible and manageable conception ; which represents though it does not explain the things and processes of nature ; this clothes the fruitful but otherwise indefinite ideas in such a form that the strict logical methods of thought can be applied, that the human

mind can in its inner chamber explore a chain of reasoning the result of which corresponds to the phenomena of the outer world. —J. T. MERTZ

48. All abstract sciences can be arranged in a hierarchy in the order Mathematics, Physics, Chemistry, Biology, Sociology in which each science is (i) historically older (ii) logically simpler (iii) more widely applicable (iv) independent of all those that follow it and will attain the positive stage much earlier. —COMTE

49. Mathematics rightly viewed possesses not only truth, but supreme beauty—a beauty cold and austere like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure and capable of stern perfection, such as only the greatest art can bestow. The true spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence is to be found in mathematics as surely as in poetry. What is best in mathematics deserves not only to be learnt as a test, but to be assimilated as a part of daily thought and brought again and again before the mind with every encouragement. Real life is to most men, a long second best, a perpetual compromise between the ideal and the possible, but the world of pure reason knows no compromise, no practical limitations, no barrier to creative activity, embodying in splendid edifices the passionate aspirations after the perfect from which all great work springs. Remote from human passions, remote even from the pitiful facts of nature, the generations have gradually created an ordered cosmos where pure thought can dwell as in its natural home and where one at least of our nobler impulses can escape from the dreary exile of the actual world. —BERTRAND RUSSEL

50. Pure mathematics is a branch of logic and all propositions of mathematics are propositions of logic.
—LOGISTIC SCHOOL

51. Pure mathematics is founded on a basic intuition of

the possibility of constructing an infinite series of numbers.

—Intuitionist School

52. Pure mathematics is the science of formal structure of symbols.

—Formalist School

*53. Mathematics derives its main strength from its following features :—

(i) Abstractness (ii) Generalisation (iii) Logical consistency (iv) Depth (v) Precision (vi) Seriousness (vii) Elegance (viii) Economy of thought (ix) Thoroughness (x) Significance (xi) Clarity (xii) Permanence.

*54. Mathematics has made three main contributions to civilization —the powerful method of deductive reasoning, the mathematical description of nature and mind, and its contribution to social sciences through the powerful tools of probability and statistics.

55. Mathematics is both the queen and the handmaiden of the sciences.

—E T. Bell

56. The development of mathematics at any time in any country is the true mirror of its civilization and progress at that time.

57. A philosopher like Fourier ought to know that the unique end of all science is the honour of the human spirit and a question pertaining to number is as pertinent thereto as any of the physical world.

—Jacobi

58. "To pure mathematics, and may it never find any use."

—A Cambridge Don

59. If mathematics is the queen of the sciences, then Arithmetic (theory of numbers) must be the queen of mathematics due to its supreme uselessness.

—Gauss

60. "What is the use of Substitution Groups ?"

"Thank God, for all I know, it has none."

—Sylvester

61. "Bessel Functions must be taught, since they are very useful," "Yes, Bessel functions are very beautiful functions in spite of their having practical applications."
—A Cambridge Discussion

62. In mathematics we contemplate absolute truths, which existed in the Divine Mind before the morning stars sang together and which will continue to exist there when the last of their radiant host shall have fallen from heaven.
—Everett

63. Pure mathematics consists entirely of such assertions that if such a proposition is true of anything, then such and such another proposition is true. It is essential not to discuss whether the first proposition is really true and not to mention what the anything is of which it is supposed to be true.
—Bertrand Russel

64. Mathematics is the logical study of shape, arrangement, structure and quantity. Pure or abstract mathematics is the study and development of the principles of mathematics as such (for their own sake and possible future applications) rather than for their immediate use. Applied mathematics is concerned with the study of the physical, biological and sociological worlds. Broadly speaking, a mathematical structure utilizing in addition to the purely mathematical concepts of space and number, the notions of time and matter, belongs to the domain of applied mathematics.
—Dictionary

65. It must not be imagined that the sole function of mathematics—the handmaiden of the sciences is to serve science. Mathematics has also been called the queen of the sciences. If occasionally the queen has seemed to beg from the sciences, she has been a very proud sort of beggar, neither asking nor accepting favours from any one of her more affluent sisters. What she gets, she pays for. Mathematics has a light and a wisdom of its own, above any possible applications to science and it will richly reward any intelli-

gent human being to catch a glimpse of what mathematics means. This is not the old doctrine of art for arts sake, it is art for humanity's sake. After, all the whole purpose of science is not technology - God knows we have gadgets enough already. We must therefore know those things which the great mathematicians have considered worthy of loving understanding for their intrinsic beauty. —E. T. BELL

66. A mathematician has not thoroughly understood his own work till he has made it so clear that he can go out and explain it effectively to the first man he meets in the street. —LAGRANGE

67. Students of mathematics are familiar with the phenomenon of 'slow development' or 'subconscious assimilation'. The first time something new is studied, the details seem to be too numerous and hopelessly confused and no coherent impression of the whole is left on the mind. Then on returning from a rest, it is found that everything has fallen in its own place with its proper emphasis like the development of a photographic film. The maxim to be followed is : Go on and faith will come to you. —E. T. BELL

68. The genesis of mathematical creation is an activity in which the human mind seems to take least from the outside world, in which it acts or seems to act only of itself or on itself, so that in studying the procedure of mathematical thought, we may hope to reach what is most essential in man's mind. —POINCARE

69. If mathematics invokes only the rules of logic such as are accepted by all normal minds, if its evidence is based on principles common to all men, which none could deny without being mad, how does it come about that so many persons are refractory to it and how is error possible in mathematics ? —POINCARE

70. In fact what is mathematical creation ? It does not consist in making new combinations with mathematical entities already known—any one could do that, but the combinations so made would be infinite in number

and most of them absolutely without interest. To create consists precisely in not making useless combinations and in making those which are useful and which are only a small minority. Invention is a discriminating choice. In the mathematical soul, ideas arise in crowds, they collide untill by inter-locking and interaction, they form stable collections.

— POINCARE

71. Often when one works at a hard question, nothing good is accomplished at the first attack ; then one takes a rest long or short, and sits down anew to the work. During the first half-hour, as before. nothing is found and then all of a sudden the decisive idea presents itself to the mind. It is very probable that this rest has been filled with unconscious work but this unconscious work is possible and certainly is only fruitful if it is on the one hand preceded and on the other hand followed by a period of conscious work. These sudden inspirations never happen except that some days of voluntary effort which might have appeared useless, but which in fact have set going the unconscious machine and without them it would not have moved and would have produced nothing.

—POINCARE

72. The great mathematicians have played a part in the evolution of scientific and philosophic thought comparable to that of philosophers and scientists.

—E.T. BELL

73. The basic ideas of modern mathematics from which the whole vast and intricate complexity has been woven by thousands of workers are simple, of boundless scope and within the understanding of any human being with normal intelligence. —E T. BELL

74. Assimilation of some of the most important and invigorating ideas of modern mathematics—groups, spaces of many dimensions, non-euclidean geometry and symbolic logic will be found as refreshing as a drink of cold water on a hot day and as inspiring as any art. —E.T. BELL

75. Mathematical truths are immortal, imperishable, the same yesterday, to-day and for ever ; the very stuff of which eternal varieties are fashioned and one glimpse of changelessness, behind all the recurrent cycles of birth, death and decay our race has ever caught.

—E.T. Bell

76. Compared to what Greece did at the height of its glory, mathematics of the nineteenth century is a bonfire beside a puny candle. —E.T. Bell

77. If some devotee (of mathematics) is reproached for spending his life on what to many may seem the foolish pursuit of a beauty having no immediate reflection in the lives of his fellowmen, he may repeat Poincare's 'Mathematics for mathematics sake'. People have been shocked by this formula, yet it is as good as life for life's sake, if life is but misery.

78. If we care to inspect the symbols in nature's great book through the critical eye of modern science, we shall perceive that we ourselves did the writing and that we used the particular script we did because we invented it to fit our own understanding.

If number rules the universe, as Pythagoreans asserted, number is merely our delegate to the throne, for we rule the number.

79. Mathematics seems to endow one with a new sense, the sublimated common sense. – G.C. Darwin

80. Mathematics is indispensable to the beginning, the middle and the end of such instruction as the duties of education require of us. —Herbert

81. The progress and the improvement of mathematics are linked to the prosperity of the State. —Napolean

82. Mathematics is the gate and key of the Sciences .. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences. And what is worse, men who are thus

ignorant are unable to perceive their own ignorance and so do not seek a remedy. —BACON

83. All scientific education which does not commence with mathematics, is of necessity, defective at its foundation. —COMTE

84. A natural science is a science only in so far as it is mathematical. —KANT

85. Mathematics is the indispensable instrument of all physical research. —BERTHELOT

86. It is almost impossible to follow the latter development of physical or general chemistry without a working knowledge of higher mathematics. —MELLOR

87. In mathematics, we find the primitive source of rationality and to mathematics must the biologists resort for means to carry on their researches. —COMTE

88. In the development of wireless telegraphy, the theory of complex numbers and differential equations bore a much larger share than steel, brass or glass. —KELVIN

89. When we emerge from qualitative side of physical phenomena and want to consider their quantitative side, mathematical knowledge is indispensable.

90. The claim of any particular branch of natural philosophy to be considered as a science can be assessed only on the amount of mathematics it uses. —COMTE

91. On the assumption that nature is mathematical, Mendel predicted the existence and properties of some elements undiscovered till then, Leverrier predicted the existence of planet Neptune unobserved till then, Maxwell predicted wireless waves which Hertz later demonstrated, Raleigh and Stokes predicted optical phenomena associated later with Raman effect.

92. It is in mathematics we ought to learn the general method always followed by the human mind in its positive researches.

93. The social sciences mathematically developed are to be the controlling factors in civilization. —WHITE

94. All great scientific discoveries are but the reward of patient, painstaking sifting of numerical data.

—KELVIN

95. Analytical and graphical treatment of statistics is employed by the economist, the business expert, the actuary etc, with the most surprisingly valuable results, while symbolic language involving mathematical methods has become a part of well-nigh every large business. —KARPINSKY

96. I have tried to show how it is possible to estimate numerically through suitable mathematical equations, the velocity of circulation of money. This problem had been pronounced insolvable and without mathematical analysis, it might well be so considered still. As a second example, both Prof. Frisch and myself by independent methods, both of them highly mathematical, have shown how to measure statistically marginal utility. This would supply a mathematical criterion by which to judge the justice of a progressive income-tax. — IRWING FISHER

97. There are few things which we know which are not capable of mathematical reasoning and when these cannot, it is a sign our knowledge of them is very small and confused and where a mathematical reasoning can be had, it is as great a folly to make use of another, as to grope for a thing in the dark when you have a candlestick standing by you. —ARTHENBOT

98. Logic has borrowed the rules of geometry. The method of avoiding error is sought by every-one. The logician professes to lead the way, the geometer alone reaches it and aside from their science, there is no true demonstration.

—PASCAL

99. Mathematics is but the higher development of symbolic logic. —WHITHAM

100. The two great components of the critical movement, though distinct in origin and following separate paths are found to converge at least in the thesis : "Symbolic logic is mathematics, mathematics is symbolic logic, the twain are one." —KEYSER

101. The value of mathematical instruction consists not in the applicability of its doctrines, but its methods. Mathematics will ever remain the most perfect type of deductive method in general. —MILL

102. Philosophers, when they have possessed a thorough knowledge of mathematics have been among those who have enriched the science with some of its best ideas. On the other hand, it must be said that with hardly an exception, all the remarks on mathematics made by those philosophers who have possessed but a slight or hasty or late acquired knowledge of it are entirely worthless, being either trivial or worthless. —WHITEHEAD

103. Mathematics and music are the most sharply contrasted fields of scientific activity which can be found and yet related and supporting each other, as if to show forth the secret connection which ties together all activities of our mind and which lead us to surmise that the manipulations of artists genius are but the unconscious expression of a mysteriously active rationality. —HELMHOLTZ

104. Music is a hidden exercise in arithmetic of a mind unconscious of dealing with numbers. —LEIBNITZ

105. The symmetry and proportion in its results, a lack of superfluity, an exact adaptation of means to ends, the indispensability of each word, each letter, each little dash, that among all artists raises the mathematician nearest to the world creator, it establishes a sublimity which is equalled in no other art—something like it exists at most in symphonic music. —BOLTZMAN

106. Mathematics too has its triumphs of the creative imagination, its beautiful theorems, its proofs and processes, whose perfection of form has made them classic. He must be a practical man who can see no poetry in mathematics. —WHITE

107. The domain of mathematics is the sole domain of certainty. There and there alone prevail the standards by which every hypothesis respecting the external universe and all observations and all experiments must be finally judged. It is the court of last resort...it is here that mind as mind attains its highest estate.

—KEYSER

108. The most distinct and beautiful statement of any truth must take at last the mathematical form.

—THOREAU

109. Of poetry and geometric truth, and their high privilege of lasting life, from all internal injury exempt, I mused.

—WORDSWORTH

110. The all-pervading utility of mathematics is not due to facts or methods, it is all pervading, as it was devised by man for that purpose.

111. The nearer man approaches to mathematics, the farther away he moves from animals. —S. GASSON

112. Even stranger things have happened, and perhaps the strangest of all is the marvel that mathematics should be possible to a race akin to the apes. —E.T. BELL

113. The universal homage paid to mathematics arises from deep human needs that can be satisfied only through mathematical activities. What is mathematics really concerned with ? Where do mathematical concepts come from ? Are mathematical ideals discovered or invented ? How did mathematics develop ? How is mathematics effected by the cultural malieu in which it is natured ? What motivates and directs the activities of a mathematician ? How do they think ? How is mathematics related to reality and to creative art ? What are some of the cultural bearings and humanistic implications of mathematics ? What is the basic

need that urges man in his mathematical activities ? These are some questions, the answers to which are of vital interest both to mathematics and civilization.

114. Scientific (mathematical) ideas have a history. They come into being in response to certain human needs, which interestingly enough are not confined to scientists. In order to appreciate the full import of these ideas, they must be examined in the light of man's over-all intellectual, aesthetic and spiritual activities.

115. The final aim of each natural science is to resolve itself into mathematics. —HELMHOLTZ

116. All the pictures which science draws of nature and which alone seem capable of according with observational facts are mathematical pictures .. The universe appears to have been designed by a pure mathematician. —JEANS

117. The aim of scientific thought is to see what is general in what is particular and what is permanent in what is transitory and in this vision science utilizes the general abstraction of mathematics and adopts the theory of invariants. —WHITEHEAD

118. With the calculus as a key, mathematics can be successfully applied to the explanation of the course of Nature. —WHITEHEAD

119. When we reach the core of physical reality, the truth is presented in mathematical equations. —RICHARD

120. The Pythagoreans, long ago held that the world was not a class, but a cosmos harmoniously ordered by invariable mathematical laws. —WEYL

121. Nature seems to know the rules of mathematics, as the mathematicians have formulated them in their studies without drawing on experience of outer world. —JEANS

122. Poetry has been called a creation, a making, a fiction,

and mathematics has been called the most stupendous of all fictions. —Hill

123. The mathematicians' best work is art, a high and perfect art as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch each other.
—Mittag, Leffler

124. Mathematics is the music of reason. The musician feels mathematics, the mathematician thinks music.
—Sylvester

125. Read your instruments and obey mathematics, that is the whole duty of the scientist. —Dantzig

126. Mathematics as a purely formal science is indeed identical with logic. —Cohen

127. Mathematics is usually recognized to be properly a branch of logic. —Bridgeman

128. Pure mathematics is the class of all propositions of the form p implies q, where p and q are propositions containing one or more variables the same in two propositions and neither p nor q contains any constants except logical constants—And logical constants are all notions definable in terms of the following ; implications, the relation of 'a term' to a class of which it is a number, the notions of 'such that', the notion of relation and such further relations as may be involved in the general notion which is not a constituent of the propositions which it considers namely the notion of truth. —Bertrand Russel

129. Mathematics is the subject in which one never knows what one is talking about, nor if what he says is true.
—Bertrand Russel

130. We shall assume that all mathematics is deductive.

131. Mathematics is the language of the possible.

132. A mathematical structure consists of certain undefined terms (symbols) certain postulates upon them (that is certain relations between them), an agreed logic (that is method of drawing conclusions) and strings of

theorems derived by means of the logic. The structure is built rather than constructed. —FORT

133. Thus behind everything that is simple, familiar and common place, in life and in Nature, there is a challenge to the creative genius of the mathematician. It is a great thing to participate in the intellectual adventure of discovering of known mathematical patterns in unfamiliar situations and of inventing new patterns for familiar situations. But how often we find in our country students and teachers of mathematics either for want of encouragement or for lack of faith and perseverance, turning to other interests and pursuits, after a short and unsuccessful love affair with mathematics. —NARLIKAR

134. As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality. —ALBERT EINSTEIN

135. It is no paradox to say that in our most theoretical moods, we may be nearest to our most practical applications. —A. N. WHITEHEAD

136. There is nothing so prolific in utilities as abstractions. —FARADAY

137. On the one hand, mathematics is a study of certain aspects of the human thinking process ; on the other hand when we make ourselves masters of a physical situation, we so arrange the data as to conform to the demands of our thinking process. It would seem probably therefore that merely in arranging the subject in a form suitable for discussion, we have already introduced the mathematics—the mathematics is unavoidably introduced by our treatment and it is inevitable that mathematical principles appear to rule Nature.

138. Modern mathematics is postulational in essence, considered as a logical deductive system, mathematics makes no pretence at interpreting the subject matter of its symbols. Thus a mathematical structure of itself possesses no meaning, nevertheless it does have

form or pattern. It is this pattern which serves the scientist so well. When the subject matter symbols of a given mathematical structure form have been appropriately particularised, the mathematics becomes a system in theoretical science and the postulates of mathematics become laws of nature. The availability of many different mathematical structures has proved unbelievably fruitful.

139. A mathematician may say anything he pleases, but physicist must be at least partially sane. —GIBBS

140. Philosophers know almost nothing about everything, mathematicians know everything about nothing, physicists know something about something.

141. To pure mathematician the proper aim of mathematical analysis is the establishment of most general conclusions from the smallest number of restrictive hypotheses. There is no element of uncertainty or approximations in his conclusions. —LINDSAY

142. The physicists are occasionally annoyed by the penchant of the mathematicians for proving that under such and such conditions, a solution of a certain equation exists without in the least indicating how to find it.
—LINDSAY

143. Mathematics is a language. —GIBBS

144. Mathematics is the language of physical sciences and certainly no more marvellous language was ever created by the mind of man. —LINDSAY

145. It seems to me that all persons who think logically at all are effectively thinking mathematically whether they are willing to admit it or not. The utilization of a special symbolism is a mere device to facilitate logical thinking and unwillingness to learn it probably reflects human inertia rather than human incompetence. Like the hero in Molier's plays, many of our contemporaries would doubtless be amazed to learn of the amount of mathematics they really know.
—LINDSAY

146. The mind of man is a strange thing, not content to economise by using the same mathematical notations to describe the most diverse phenomena, it feels it gains a deeper insight into one particular section of experience by describing it with a wide variety of mathematical methods. It is much more illuminating to solve one problem by two or more different methods than to solve any number of problems by the same mathematical method. —LINDSAY

147. Mathematics is the art of saying the same thing in many different ways. —MAXWELL

148. 'Real' mathematics must be justified as art, if it can be justified at all ; it has no other defence. Like poetry or music, it sustains a lofty habit of mind. —HARDY

149. The pure mathematics of today will be the physics of tomorrow. The pure mathematician creates his own problems, the mathematical physicist has to take the problem which nature provides —he cannot dodge them.

150. As long as man retains his curiosity about his environment, he will try to describe nature and for this he will continue to use mathematical reasoning. This will remain true whether we are concerned with metrical or non-metrical aspects of experience. Of one thing, one may be sure ; physics without mathematics will for ever be incomprehensible.

151. Since mathematics is an entirely free activity, unconditioned by external world, it is more just to call it an art than a science. Although it can be used to understand natural phenomena, it is subjective, a product of free creative imagination. The mathematician who said that he was less interested in results than in the beauty of the methods by which he found the results was not expressing an unusual sentiment. —SULLIVIN

152. It is certain that the real function of art is to increase our self-consciousness, to make us more aware of what we are, and therefore what the universe in which

we really live is. And since mathematics in its own way, also performs this function, it is not only aesthetically charming but profoundly significant. It is an art and a great art. —SULLIVIN

153. It appears that mathematical ideas are arranged somehow in strata ; the ideas in each stratum being linked by a complex of relations both among themselves and with those more above and below. The lower the stratum, the deeper and in more general more difficult the idea. Thus the idea of an irrational number is deeper than that of an integer. — HARDY

154. It is argued that mathematics is useful, it is a powerful aid to logical thinking, it at least seems to be correct and that it never loses its creations. —SHAW

155. When we reach the sphere of mathematics, we are among the processes which seem to some the most inhuman of all human activities, the most remote from poetry. Yet it is here that the artist has the fullest scope for his imagination. We are in the imaginative sphere of art and the mathematician is engaged in a work of creation which resembles music in its orderliness and yet reproducing on another plane the order of universe and so becoming, as it were, a music of the spheres. The mathematician has reached the highest rung of human thought.

—HAVELOCK ELLIS

156. Mathematics is engaged, in fact, in the profound study of art and the expression of beauty. The medium used is very ethereal, being pure ideas, nothing material and this justifies the statement that mathematics is subtle fine art. Its material is sublime to the limit.

—SHAW

157. Mathematics is on the artistic side, a creation of new rythms, orders, designs and harmonies and on the knowledge side is a systematic study of the various rythms, orders, design and harmonies. Mathematics is on one side, the qualitative study of structure of

beauty and on the other side is a creator of new artistic forms of beauty. The mathematician is at once creator and critic, not always of course in the same person. —SHAW

158. The history of mathematics is exhilarating, because it unfolds before us the visions of an endless series of victories of the human mind, victories without counter balancing dishonourable and humiliating failures and without atrocities. I believe on the basis of my historical experience that the mathematics of the twenty-fifth century will be as different from that of today as the latter is from that of the sixteenth century. —SARTON

159. The history of mathematics should really be the kernel of the history of civilization. —SARTON

160. The main sources of mathematical invention seem to be within man rather than outside of him, his own invetrate and insatiable curiosity, his constant itching for intellectual adventure and likewise the main obstacles to mathematical progress also seem to be within himself, his scandalous laziness and inertia, his fear of adventure, his need of conformity to old standards and his obsession by mathematical ghosts. —SARTON

161. The essence of mathematics lies in its freedom. —GEORGE CANTOR

162. It has been said that the pure mathematician is never as happy as when he does not know what he is talking about. —W. F. G. SWANN

163. Here then in mathematics, we have a universal language, useful, intelligible everywhere in place and in time. Here is a discipline of a hundred branches, fabulously rich, literally without limit in its sphere of applications, laden with honours for an unbroken record of magnificent accomplishment. Here is a creation of the mind, both mystic and pragmatic in appeal, austere and imperious as logic ; it is still sufficiently sensitive and flexible to meet each new need. Yet this vast

discipline rests on the simplest and most primitive foundations wrought by imagination and logic out of a handful of childish rules. —KASTNER AND NEWMAN

164. That mathematics enjoys a prestige unequalled by any other flight of purposive thinking is not surprising. It is at once so indispensable in practical affairs and so easily the masterpiece of pure abstractions that the recognition of its pre-eminence among man's intellectual achievements is no more than its due.

165. We have overcome the notion that mathematical truths have an existence independent and apart from our own minds. Today mathematics is unbounded, it has cast off its chains. It is recognized to be as free as the wind, as prehensile as imagination. Non-Euclidean geometry is proof that mathematics unlike the music of the spheres is man's own handwork subject only to limitations imposed by laws of thought.

166. While those who oppose logical positivism grant that it serves some purpose, they assail the stultification of discussion, the narrowing of horizons which it inevitably entails. We share the feeling that mathematics is more than a factory of tautologies, rather it is a valuable aid to carry on the highest aspirations of the creative intellect.

167. The development of mathematics is a counterpart of the everlasting struggle for greater comprehensiveness and greater freedom from the particular to the general, from configurations bounded by straight lines to pathological curves, from the property of this or that figure to property of all figures, from one dimension to n-dimensions, from the finite to the infinite. In this march imagination has played a notable role.

168. Mathematics is an activity governed by the same rules as are obeyed by the symphonies of Beethoven or by the paintings of De Vinci and poetry of Homer. Ultimately mathematics reaches pinnacles as high as those attained by the imagination in its most daring reconnoitres.

169. In mathematics we do nothing but substitutions.

170. All great battles of mathematics and physics are fought on fields of characteristic zero. —WEYL

171. It is India which gave us the ingenius method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value, a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity, the great ease which it lent to all the computation, puts our arithmetic in the first rank of useful inventions and we appreciate the grandeur of this achievement the more when we remember that it escaped the genuis of Archimedes and Apollonius, two of the greatest men produced by antiquity. —LAPLACE

172. All results of the profoundest mathematical investigation must ultimately be expressible in simple form of properties of integers. —KRONECKER

173. As the prerogative of natural science is to cultivate a taste for observation, so that of mathematics is almost from the starting point, to stimulate the faculty of invention. —SYLVESTER

174. (Transfinite arithmetic) On whole is that, whole (too) is this ; from whole, whole cometh ; take whole from whole, whole remains. —ISA UPANISHAD

175. Cases could be cited where physicists have been led astray by inattention to mathematical rigour but these are rare compared with mathematicians' misadventures through lack of physical insight. —EDDINGTON

176. I think the idea that the purpose of study is to arrive at a string of proofs of propositions is a little overdone even in pure mathematics. We might indeed say that whereas for the mathematician, insight is one of the tools and proof the finished product ; for the physicist proof is one of the tools and physical insight is the finished product. —EDDINGTON

177. Life and Cosmos are both continued fractions and therefore both are irrational.
— E. T. Bell

178. There cannot be a language more simple, more free from errors and obscurities (than of mathematics) that is to say more worthy to express the relation of natural things. It brings together the most diverse phenomena and discovers the hidden analogies which unite them.
—Fourier

179. When Napolean asked Laplace whether in his monumental 'Mecanique Celeste' there was any reference to the Deity, he is said to have replied "Sir, I have no need of that hypothesis." On hearing Napolean recount this story, Larange remarked "That, Sir is a wonderful hypothesis."

180. Guided only by their feeling of symmetry, simplicity and generality and an indefinable sense of fitness of things, creative mathematicians are inspired by the art of mathematics than by the prospect of immediate utility in science and technology.

181. Mathematics represents perhaps the highest efflorescence of the creative human spirit, rivalling the fine arts in its aesthetic appeal, excelling poetry in its condensed richness and akin to upanishdic thought in its all-embracing abstractness. It is therefore a touchstone of intellectual and moral maturity and a nation otherwise sound but whose mathematics is not up to the mark is like a good-looking and well-dressed person without character. The surest way to commit suicide is to neglect mathematics.
—A. Narasinga Rao

182. Pure science is the seed of applied science and to neglect pure sicence in a thing of this kind would be like spending a very large amount in manuring and ploughing the land and then to omit the sowing of any kind.
—J.J. Thomson

183. The type of mathematics found in any major culture is a clue to the distinctive character of the culture

taken as a whole. The history of mathematical thought is related with the history of civilization.

—KEYSER

184. A mathematician works for the intellectual joy of creations. He enjoys the harmonious beauty of his subject and the moment of successful thinking.

185. As are the crests on the heads of peacock, as are the gems on the heads of the snakes, so is the ganit at the top of sciences known as the vedanga.

—VEDANGA JYOTISA

186. The zero which gives to airy nothing, not merely a local habitation and a name, a picture, a symbol, but helpful power is the characteristic of the Hindu race from where it sprang. No single mathematical creation has been more potent for the general on go of intelligence and power.

—HOLSTEAD

187. In Chicago in 1933-34 was displayed the tree of knowledge. The branches of the tree were shown as economics, sociology, engineering etc. The roots were physics, chemistry etc. and the tap root was mathematics. No plant can grow without tap root ; so also the applied sciences which are like the flower of the plant cannot flourish unless the basic subject of mathematics develops vigorously.

188. The fact that changes in our material universe can be predicted – that they are subject to mathematical law is the most significant thing about it, for mathematical law is the concept of the mind and from the existence of the mathematical law we infer that our minds have discovered something akin to themselves *i.e.* in or behind the universe.

—WHITEHEAD

189. The study of nature, which was formerly one of the main sources of great mathematical problems, seems in recent years to have borrowed from us more than it has given us.

—ANDRE WEIL

190. One should study mathematics because it is only

through mathematics that nature can be conceived in harmonious form.

—BIRKHOFF

191. Thus, here again (in the theory of quasi-analytic functions) as so often in my work, the motivation which has led me to the study of practical problem has also induced me to go into one of the most abstract branches of pure mathematics.

—N. WEINER

192. The book of nature is written in the characters of geometry.

193. The crucial steps in the intellectual adventure of solving problems are—

(i) a dive from the world of reality into the world of mathematics.

(ii) a swim in the world of mathematics.

(iii) a climb from the world of mathematics back into the world of reality carrying the prediction in our teeth.

—SYNGE

194. The mathematician invents, so that the scientist may discover.

195. That nature is mathematical was known to the Greeks; the complementary fact that mathematics is natural is however just beginning to be grasped. These two aspects are so intimately related as are the two sides of a coin.

—BIRKHOFF

196. The surprising thing is that the scientist and the mathematician can collaborate and establish isomorphisms between thought and nature.

—V. V. NARLIKAR

197. Mathematics is a method of inquiry known as postulational thinking. The method consists in carefully formulating definitions of the concepts to be discussed and in explicitly stating the assumptions that shall be the basis of reasoning. From these definitions and assumptions conclusions are deduced by the application of the most rigorous logic man is capable of making.

—MORRIS KLINE

198. Mathematical language is precise, so precise that it is often confusing to people unaccustomed to forms.
—Morris Kline

199. Mathematical style aims at brevity and formal perfection. It sometimes succeeds too well and sacrifices the clarity its precision seeks to guarantee.
—Morris Kline

200. Mathematics is more than a method, an art, and a language. It is a body of language with content that serves the physical and social scientist, the philosopher, the logician, and the artist ; content that influences the doctrines of statesmen and theologians, content that satisfies the curiosity of the man who surveys the heavens and the man who muses on the sweetness of musical sound; and content that has undoubtedly, if sometimes imperceptibly shaped the course of modern history.
—Morris Kline

201. Mathematics has indeed been a beacon light to the sciences and has continually helped them in reaching the position they occupy in our present civilization. It is even correct to assert that modern science triumphs by virtue of mathematics.
—Morris Kline

202. Mathematics is a spirit of rationality. It is this spirit that challenges, stimulates, invigorates and drives human minds to exercise themselves to the fullest. It is this spirit that seeks to influence decisively the physical, normal and social life of man, that seeks to answer the problems posed by our very existence, that strives to understand and control nature, and that exerts itself to explore and establish the deepest and utmost implications of knowledge already obtained.
—Morris Kline

203. Mathematics is a living plant which has flourished and languished with the rise and fall of civilizations.
—Morris Kline

204. Do not imagine that mathematics is hard and crabbed and repulsive to commonsense. It is merely the etherealization of commonsense. —Lord Kelvin

205. Mathematics deals with the investigation of concepts which express the relation of objects to one another.
—PAPPERITZ

206. Mathematics is engaged solely in tracing out the consequences of hypotheses.
—PIERCE

207. Mathematics is a pure system in the sense of a formal theory. It is a science (W) which has the following properties :

(a) The propositions in W contain in addition to purely logical concepts, only variables. (b) No proposition in W is a proposition of pure logic. (c) There exists a finite subset A of W (called the initial propositions of W) such that the propositions of A are compatible with and independent of one another, and from these every other proposition in W can be deduced by purely logical deductions. Every proposition, finally, which can be deduced from A is a proposition in W unless it belongs to pure logic. —A. KORSETT

208. There is an astonishing imagination in the science of mathematics. We repeat there was far more imagination in the head of Archimedes than in that of Homer.
—VOLTAIRE

209. It may well be doubted whether in all the range of science, there is any field so fascinating to the explorer, so rich in its hidden treasures, so fruitful in its delightful surprises, as that of pure mathematics. The charm lies chiefly in the absolute certainty of its results, for that is what beyond all mental treasures the human intellect craves for... Science has little else to offer that will meet the demand of its votaries than the conclusions of pure mathematics. —DODGSON

210. I will not go so far as to say that to construct a history of thought without profound study of mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. This will be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. The simile is singularly exact. For Ophelia is quite essential to

the play. She is very charming and a little mad. Let us grant that the pursuit of mathematics is a driving madness of the human spirit, a refuge from the goading urgency of contingent happenings.

—WHITEHEAD

211. If one be bird-witted, that is easily detracted and unable to keep his attention as long as he should, mathematics provides a remedy ; for in them, if the attention be caught away but a moment, the demonstration has to be commenced anew. —BACON

212. The universe is the grand book of philosophy. The book lies continually open to man's gaze, yet none can hope to comprehend it who has not first mastered the language and the characters in which it has been written. This language is mathematics, these characters are triangles, circles and other geometrical figures.

—GALLILEO

213. Behind the artisan is the chemist, behind the chemist is the physicist, behind the physicist is a mathematician. —WHITE

214. Mathematics is no more the art of reckoning and computation than architecture is the art of making of brick or hewing of wood, no more than painting is the art of mixing colours, no more than the science of geology is the art of breaking rocks or the science of anatomy the art of butchering. —KEYSER

215. Are we to be surprised if a set of postulates each of which has been formed from experience should as a whole be isomorphic with some portions of experience and useful in studying it ? Why should we marvel if space is non-Euclidean, or if complex numbers are useful in studying alternating currents and what not ? I shall never be surprised when a well-built mathematical system is applied to any branch of science. It seems absurd to say, "Nature is mathematical." Rather man's mathematics is based on man's nature.

—FORT

216. The study of mathematics has a great cultural and

educative value, since it inculcates regard for truth, intellectual honesty, exactness, clarity and precision. Mathematics seems to have been wrought into the very fibre of man's mind and its development and manifestation are in direct proportion to the degree of civilization which man attains. Mathematical methods are the simplest which man knows and its results are the most dependable the world has ever found.

—Ram Behari

217. Logic and mathematics both develop as our understanding and knowledge of the world around us increases. Our processes of logical thought must be looked upon as the nervous pattern of the human species at a certain stage of its development, a pattern which develops as the species itself develop.

—H. J. Bhabha

218. A mathematician works for the intellectual joy of creation. He enjoys the harmonious beauty of his subject and the triumphant moments of successful thinking. A mathematician creates ideal situations and builds up coherent and consistent systems. All that he is concerned with is internal consistency.

—Ram Behari

219. Mathematics today is the instrument by which the subtle and new phenomena of nature that we are discovering can be understood and coordinated into a unified whole. In this some of the most advanced and newest branches of mathematics have to be employed and contact with an active school of mathematics is therefore great asset for theoretical physicists.

—H. J. Bhabha

220. This abstraction (of mathematics) has affected civilization a great deal. People have been furnished with a tool which enables them to become more and more explorative in the attempt to harness nature.

—Ram Behari

221. Classics and humanities have been studied as a discipline, as a liberal education. Science also is a discipline

but mathematics being both an art subject and a science subject is an ideal discipline.

—Ram Behari

222. Our experience hitherto justifies us in believing that Nature is the realization of the simplest conceivable mathematical ideas.

—Ram Behari

223. The modern scientific man is only talking about certain mathematical relations when he talks about the entities out of which he intends to construct the universe.

—Sullivan

224. Man tries to make for himself, in the way that suits him best, a simplified and intelligible picture of the world and thus to overcome the world of experience for which he tries to some extent to substitute the cosmos of his.

—Einstein

225. When followed in the proper spirit, there is no study in the world which brings into more harmonious action all the faculties of the mind than mathematics. There is none other which prepares so many agreeable surprises for its followers, more wonderful than the transformation scene of a pantonime, or like this seems to raise them, by successive steps of initiation to higher and higher states of conscious intellectual being.

—Sylvester

226. Pure mathematics contains within itself the cause of its being and its methods of proof. For in complete independence mathematics creates for itself the object of which it treats, its magnitudes and laws, its formulas and symbols.

—Dillimann

227. Mathematics is perfectly free in its development and it is subject only to obvious consideration that its concepts must be free from contradictions themselves, as well as definitely and orderly related by means of definitions to the previously existing and established concepts.

—Cantor

228. In most sciences one generation tears down what another has built. In mathematics alone each generation builds a new storey to the old structure. —HERMANN

229. In mathematics sophistry had no place from the beginning. Mathematicians having had the wisdom to define accurately the terms they use. Accordingly we find no parties among mathematicians and hardly any disputes. —REID

230. Mathematical analysis is co-extensive with nature itself ; it is a difficult science which forms but slowly but preserves carefully every principle once acquired, it becomes stronger incessantly amidst all the changes and errors of the human mind. It compares the most diverse phenomena and discovers the secret analogies which unite them, it explains them in the language as if in witness to the unity and simplicity of the plan of the universe and to make more manifest the unchangeable order which presides over all natural causes. —FOURIER

231. In mathematics we behold the conscious logical activity of the human mind in its purest and most perfect sense. —HELMHOLTZ

232. We cannot get more out of the mathematical mill than we put into it, though we may get it in a form infinitely more useful for our purpose. —HOPKINSON

233. Just as the astronomer, the physicist, the geologist look about in the world of sense, so the mind of the mathematician goes forward in the universe of logic...exploring the heights and depths for facts, ideas, classes, implications, relations and the rest. —KEYSER

234. The reasoning process in mathematics is not different from that required in other branches of knowledge, it it simply more intense and requires quickness of mind in perceiving logical sequences, love of order, methodical arrangement, and harmony, distinctness of conception. —PRICE

235. Mathematics, though abstract, is not detached from life. It is just the ideal handling of the problems of life, as sculpture may idealise a human figure or as poetry or painting may idealise a figure or a scene.
—KEYSER

236. The world of ideas which mathematics discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connection of its parts are the surest grounds of its title for human regard. —SYLVESTER

237. Mathematics, once fairly established on the foundation of a few axioms and definition, as upon a rock, has grown from age to age so as to become the most solid fabric that human reason can boast of. —REID

238. No one will venture to estimate lightly the debt which the world owes to mathematicians in that they treat in their own language matters of utmost importance.
—GOETHE

239. Many arts there are which beautify the mind of man ; of all these none do more garnish or beautify it than those arts which are called mathematical.
—BELLINGSLEY

240. As the sun eclipses the stars by his brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them.
—BRAHMPUTRA

241. What science can there be more noble, more excellent, more useful for men, more admirable, high and demonstrative than that of mathematics.
—BENJAMIN FRANKLIN

242. Mathematics is a type of thought which seems ingrained in the human mind, type of thought so essentially characteristic of the human mind, so little influenced by environment, so uniformly present in all civilizations. —YOUNG

243. The nineteenth century which prides itself upon the invention of steam and evolution, might have derived more legitimate title to fame from the discovery of pure mathematics. —Russel

244. Everybody praises the incomparable power of the mathematical method, but so is everybody aware of its incomparable unpopularity. —Rosanes

245. The mathematical method is the essence of mathematics. —Fovalis

246. One may be a mathematician without being able to compute—It is possible to be a great computor without the slightest idea of mathematics. —Novalis

247. The true mathematician is always a good deal of an artist, an architect, a poet. Beyond the world, though perceptibly connected with it, mathematicians have intellectually created an ideal world which they attempt to develop into the most perfect of all worlds and which is being explored in all directions.
—Pringsheim

248. Mathematics has beauties of its own – a symmetry and proportion in its results, a lack of superfluity, an exact adaptation of means to ends, which is exceedingly remarkable and to be found elsewhere only in the works of greatest beauty. When this subject is properly presented, the mental emotion should be that of enjoyment of beauty, not that of repulsion from the ugly and unpleasant.

249. Mathematics and poetry are the utterances of the same power of imagination, only that in the one case it is addressed to the head, and in the other, to the heart.
—Hill

250. Applied mathematics is like wine which becomes pure in course of time. —Dantzig

251. The first essential bond between mathematics and the arts is found in the fact that discovery in mathematics

is not a matter of logic. It is rather the result of mysterious powers which no one understands and in which unconscious recognition of beauty must play an important part. Out of an infinity of designs, a mathematician chooses one pattern for beauty's sake and puts it down to earth. —MORSE

252. The mathematician's patterns, like the painter's or poet's must be beautiful. The ideas like the colours or the words must fit together in a harmonious way—there is no permanent place in the world for ugly mathematics. —HARDY

253. The 'elegance' of a theorem is directly proportional to the effort it takes to see them. —POLYA

254. I am like a tailor who makes coats for his aesthetic satisfaction. Sometimes I make them with five sleeves, sometimes with seven. When it pleases me I make a coat with two sleeves, and if it fits someone I am happy enough to have him wear it. —EILENBERG

255. Mathematics is today for people who like to climb intellectual mountains.
—NEW WORLD OF MATHEMATICS

256. Mathematics today is the living language of science. It used to be taught as a dead language, as a discipline. It is more than that, it is today and for the future the most vital of new languages.
—NEW WORLD OF MATHEMATICS

257. Perhaps the leaders of the next generation will understand mathematics as a universal language even if they need not apply it.
—NEW WORLD OF MATHEMATICS

258. When mathematics is taught, it is presented mainly as a collection of slightly related techniques and manipulations. The profound yet simple concepts get little attention. If art appreciation were taught in the same way, it would consist of learning how to clip stone and mix paints. —NEW WORLD OF MATHEMATICS

259. Mathematics papers are compressed to the limit until all intuitive ideas are squeezed out. At a result one mathematician complains, "Most mathematics papers are read only three times - once by the author, once by the editor and once by the reviewer."

—New World of Mathematics

260. Symbolism offers rewards as well as perils. It is difficult to see how most modern mathematical ideas could have been developed without the marvellously compact notation in use today. Throughout history, improvements in notations have closely paralleled progress in mathematics. —New World of Mathematics

261. It was during the dark ages, while mathematics in Europe and Egypt stagnated that the Hindus developed algebra. They invented the numbers we call Arabic, but their most important contribution was the positional notation. They also started using special symbols for unknown quantities.

—New World of Mathematics

262. Today the fastest growing and most radically changing of all the sciences is mathematics. It is the only branch of learning in which all the major theories of two thousand years ago are still valid, yet never before there has been such a flood of fresh ideas.

—New World of Mathematics

263. Mathematicians have always been partial to abstractions and for its own sake. Aesthetics and not usefulness is their criterion. Yet today they are moving to heights of abstraction their forefathers hardly dared contemplate. They conceive some geometries of an infinite number of dimensions and others whose size is meaningless. They develop algebras that transcend the familiar rules of arithmetic. They explore universal abstract patterns that link such disparate concepts as numbers, space, motions and algebraic formulae. Out of all this may come one to the most exciting intellectual adventures in history.

—New World of Mathematics

264. The axioms of mathematics are by no means arbitrary for while mathematicians have pushed their abstraction beyond the physical world, they are responsible to mathematical reality—a sort of Platonic ideal that raises mathematics for above the level of a mere game —and a skilful mathematician chooses his axioms to reveal that peculiar reality. Yet mathematicians can never be sure that what they create is logically perfect, for it is beyond the power of mathematics to prove its own consistency. —NEW WORLD OF MATHEMATICS

265. Perhaps the outstanding characteristic of mathematicians is they like to do their own work. Group research does not attract them. Relatively few research papers have more than one author and almost none have more than two. Yet good mathematics can thrive in a community of scholars.
—NEW WORLD OF MATHEMATICS

266. The one attribute that virtually all creative mathematicians share is youth. Most great mathematicians have done their best work before they are forty or fifty. After fifty they generally concentrate on the teaching of philosophy or applications of mathematics to other fields. —NEW WORLD OF MATHEMATICS

267. Mathematics in the making is not a deductive science ; it is an inductive, experimental science and guessing is the experimental tool of mathematics. Mathematicians, like other scientists, formulate their theories from haunches, analogies and simple examples. They work out their rigorous proofs only after they are pretty confident that what they are trying to prove is correct and in writing these they use only the bulldozer of logical deduction. It is contended that shrewd guessing can and should be taught.
—NEW WORLD OF MATHEMATICS

268. All mathematicians, pure or applied, are keenly aware that mathematics has a reality of its own. It is a reality that is defined in the human mind and need not be confirmed by the interpretation of physical experiments. The physicist is fettered, to whatever his

instruments tell him is real ; the mathematician, however, is free to explore a much richer world mostly of his own making. In this, freedom is the chief thrill of being a mathematician. Most research mathematicians would agree with Jacobi, who, when asked why he did mathematics, replied—to glorify the human intellect. —NEW WORLD OF MATHEMATICS

269. The essence of applied mathematics is to know what to ignore. —WHITEHEAD

270. The study of mathematics consists in the acquisition of useful reflexes and in that of independent habits of thought. The acquisition of useful reflexes should never be separated from the perception of their usefulness. —A. WEIL

271. Rigour is to the mathematician what morality is to man. —A. WEIL

272. If you take the intuitionist position, you are safe. But you can prove hardly anything with their restricted reasoning. We want to make progress, so we go on using the same old unsafe logic. We have not yet run into a major paradox, although we cannot be sure we won't some day. But if we do run into a paradox, we can probably save the structure of mathematics by patching it. —GLEASON

273. It has become evident that, so far as the science of physics is concerned, we do not require to know the structure of the entities that we discuss, but only their mathematical structure. And, in truth that is all that we know. —SULLIVAN

274. Mathematics may be defined as the science of abstract form. The discernment of structure is essential no less to the appreciation of a painting or a symphony than to understand the behaviour of a physical system ; no less in economics than in astronomy. Mathematics studies order abstracted from the particular objects and phenomena which exhibit it and in a generalised form. —HARVARD COMMITTEE OF 1945

275. There is little doubt that man's rapid progress in recent decades in the control and understanding of nature, in providing himself abstract tools of creation that border upon the miraculous, and in his actual comphrehension of the powers and limitations of the human mind is a direct consequence of mathematical triumphs of the last few centuries...Thus scholarly endeavour in virtually all areas of human activity requires to an increasing extent a knowledge of mathematics and the ability to use it ; except for the study of language, mathematics may well be the most basic component of a so-called general education.

—Eves and Newsom

276. The mathematician may be compared to a designer of garments, who is utterly obvious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago ; to this day, a shape will occasionally appear which will fit into the garment as if the garment has been made for it. Then there is no end of surprise and delight. – Dantzig

277. The physicists are very proud of their subject ; the only class of people to whom they bow in respect is the class of pure mathematicians.

278. Although the point that mathematics has broad and fruitful applications has been conceded in the physical sciences and to some extent in the biological sciences, there are still many skeptics as far as the social sciences are concerned. To some extent this skeptism is due to the very legitimate objection that the social sciences are vastly more complex than the physical or biological sciences. However, this seems to indicate only that their mathematical techniques will have to be more sophisticated as well. It also indicates that the time which it will take to develop nontrivial models for social sciences may have to be substantial even in the present age of rapid scientific progress.

—Kemeny and Snell

279. Mathematics is best viewed as the study of abstract relations in the broadest sense of that word. From that point of view, it is not surprising that mathematics is applicable to any well-defined field. Whatever be the nature of the phenomena studied in a given social science, their various components do bear relations to each other, and once one succeeds in formulating these abstractly and precisely, one is in a position to apply the full machinery of mathematical analysis. Of course it is to be expected that often the mathematical model so formed will be one not studied previously by mathematicians. Therefore one may look forward to the day when the social sciences will be as major a stimulus for the development of new mathematics as physics has been in the past.

—KEMENY AND SNELL

280. The introduction of the use of mathematics in the study of physics enriched both disciplines. For some years scholars have believed that a similar conjuctive symbiosis is being or can be effected in the social studies.

—JAMES CHARLEWORTH

281. The economic world is a misty region. The first explorers used unaided vision. Mathematics is a lantern by which what was dimly visible now looms up in firm bold outline. The old phantasmagoria disappears. We see better, we see further.

—IRWING FISHER

282. Specifically one of the major uses of mathematics is in developing and clarifying points of economic theory. Obviously those who have little use for any sort of theorising will not appreciate a tool designed to help in theorising.

—L. HURWITZ

283. Gibbs had said, "Mathematics is a language." We find many examples where mathematics is a great deal more than a language. Mathematics is also a powerful instrument for the solution of some central problems of economics.

—L. HURWITZ

284. Three aspects of the mathematical method seem to stand out : the use of symbolism, axiomatic structure, a treasures of accumulated techniques and results. The idea that these aspects of mathematical method are only applicable to quantitative phenomena is by no means dead. Actually modern mathematics has the apparatus and flexibility required for dealing with what one may view as 'qualitative' phenomena. At present, methods have been developed for meaningful numerical characterisation of psychological phenomena.

—L. Hurwitz

285. The only sensible approach to discuss the limits of mathematics in economics is to let mathematics be used to the fullest extent and to observe results...The point whether there is no further room for more mathematics in economics does not arise because both mathematics and economics are growing and we cannot foresee the entire further developments in mathematics and economics—perhaps altogether new mathematics has to be invented in order to cope with the manifold forms of economic problems.

—O. Morgenstern

286. Once the nature and power of mathematics is understood, the only possible statement is that, as far as we understand mathematics now, there is nothing in the nature of that science which would exclude its decisive use in economics because of some peculiarities of the latter that mathematics could not overcome in principle.

—O. Morgenstern

287. The fact remains that, until very recently, in general professional mathematicians have shied away from economics ; non-professional workers 'applying' mathematics could hardly have been expected to produce startling results. If the economists had merely stated the problem, seen its difficulty, called for help, mathematicians might have been attracted much earlier to assume the task of inventing the proper mathematics. The purpose of mathematical economics is, of course, primarily to advance economics, not to

apply mathematics merely because some mathematics exists and we have learned that science to some extent.
—O Morgenstern

288. A feature that must be expected in any true mathematization is that the mathematical theory must yield the same insight which can be obtained from commonsense. Thereafter it gives results which go far beyond commonsense, results which commonsense would never even guess at. When this happens, some of the new results may be translable into ordinary language, but for still others, this will become impossible. When that point has been passed, a higher state in the development of a science has been reached. It is only attainable by means of mathematics.
—O. Morgenstern

289. The axiomatic method consists of formulating a set of propositions which must fulfil certain conditions. They must, in particular be free of contradictions, and the deductions derived from them must contain our knowledge of the field and beyond this hopefully lead to new insights. The axiomatic method is simply a superb technique for summarising our knowledge in a given field and for finding further knowledge deductively. The axiomatic methed tends itself especially to those phases of a science where a clear understanding of the basic concepts has been reached. Axiomatics do not burst upon the science unprepared.
—O. Morgenstern

290. The principal stimulus to mathematics has so far come from the natural sciences. The laws of nature are written in the language of mathematics, as the ancients already understood. The laws of society will be written in the same language ; that is what the moderns will have to understand. There is every indication that in the future mathematics will receive a tremendous stimulus from the social sciences. Just as mathematics has profited from having been tied so closely to the physical sciences, mathematics will

benefit from becoming deeply involved with the problems of the social world. —O. Morgenstern

291. While the need for specialisation does not present any great problem to the pure mathematician, for he has no objection to working in a restricted field, the impossibility of fully comprehending the whole range of mathematical thought does, to some extent, affect the outlook of the applied mathematician.
—J. G. Oldroyd

292. Applied mathematics is an intellectual adventure in which are combined creative imagination and authentic canons of beauty and fitness ; they combine to give insight into the nature of the world of which we ourselves and our minds are parts. —C. A. Coulson

293. It is sometimes stated that there can be no beauty in mathematics that is not completely unrelated with physical reality, that there can be no artistic value in mathematics inspired by such mundane things as the motion of the planets, the flow of a glacier or the conditions inside the sun, there can be no beauty in mathematics which leads to greater understanding of the structure of a molecule or to the development of a new principle of textile spinning. It would however indeed be surprising if the inspiration for creative work in any medium could come only from the work of other artists and from no external source ; in mathematics, at any rate, it is quite false to suppose that this is so. The creation of any physical theory can cause great emotional satisfaction to the author and its perfected formulation is worthy of being recorded in the literature to be read with pleasure by the future generations of mathematicians irrespective of its immediate usefulness. —J. G. Oldroyd

*294. Mathematics is a dynamic intellectual enterprise. It is growing at an exponential rate doubling itself in a period of ten or twelve years, and it is an intellectual activity of the highest order. While it is good to be able to enjoy mathematics created by

others, the real enjoyment of mathematics comes when one ceases to be an observer of this adventure, but joins this adventure by becoming a creative mathematician oneself.

*295. Anybody can construct a mathematical structure starting with any set of consistent axioms and postulates but only those structures survive which are significant either because they can help in explaining natural or social phenomena or they are productive of further structures which themselves are significant.

*296. In the earlier stages abstractions in mathematics arose as a result of great deal of experience, intuition, trial and error. However in more recent times abstraction has been done for its own sake deliberably and with a purpose and disciplines of abstract algebra, functional analysis etc. have developed out of such efforts.

*297. The surprise is not in the drastic assumptions we make regarding departure from reality in making mathematical models of physical situations, but in the fact that inspite of these assumptions, the observations in nature agree closely with the predictions which we carry in our teeth when we climb from the world of reality into the world of mathematics.

*298. The applied mathematician has to bring to the notice of the pure mathematicians the structures in engineering and sciences for which no isomorphic pure mathematical structures exist. Similarly he has to tell the engineer about the new structures in pure mathematics which may be useful to him. His role is that of a retail seller who has to tell the consumer what commodities are being manufactured and to inform the manufacturer about these commodities for which there is demand. He stimulates activity on both sides and of course himself benefits in the

process. His role is similar to that of the commentators of the Vedas and the Upanishadas who put the lofty teaching of these books in a form suitable to the layman. The role is not very attractive, but it is nevertheless very useful.

299. Mathematics cannot be made of definitions and their obvious consequences. In fact it is kept alive by theorems which are deductions which are not an immediate consequence of the hypotheses from which they are derived. For example, the Pythagorous theorem is a theorem because the Euclidean axioms do not appear to be concerned with right angles. It is the duty of all pure mathematicians to discover theorems. There is no known recipe for finding theorems. One approach is simply to go about constructing a theory until some problem occurs and one wants to know something which cannot be easily deduced from the hypotheses. This is the sign that a theorem is at hand. Another way of deducing theorems is to devise an ingenious system which has not been studied previously. Of course, for a good theory there must be some structure built into the hypotheses.

—Louis de Brange

300. In both pure and applied mathematics inductive and deductive processes alternate. The pure mathematician uses his experience of other mathematical structures and his intuition to formulate new axioms and postulates inductively and then uses the deductive method to prove theorems from these postulates. The applied mathematician forms hypotheses inductively on the basis of his experience, his knowledge of other pure and applied mathematical structures, his intuition and the experimental results which may be tested experimentally.

301. Recent advances in economic theory have made a good basic training in mathematics absolutely essential. It is obvious that there are some branches of the subject where mathematics is unavoidably

necessary and that a knowledge of mathematics makes precise thinking and expression possible in all branches of the science. What is more important is that the mathematical language has become a common language among economists outside India and it has made communication amongst economists easier. The Indian economist who insists on pursuing a purely literary course will not be able to put himself in the stream of new economic thinking.

—B. DUTTA

302. That mathematics has penetrated physics, chemistry, biology and sociology is a tribute to its strength, but that it has still not penetrated very deeply is a sign of its weekness, but there is no doubt that mathematics is penetrating deeper and deeper and the rate at which it is penetrating is becoming faster.

*303. In recent times, the scope of applied mathematics has become much wider and its thinking has become much more daring. While the earlier applied mathematicians had faith that mathematics is useful in some situations, the article of faith with the present applied mathematicians is that there is no situation in which mathematics cannot be applied. When we reach the core of physical reality, the truth must appear in a mathematical form. Mathematics is already being applied in controlling stocks, in determining recruitment policies for staff, in replacement of old machinery, in allotment of resources for optimum production, in determining how facilities for service should be increased to meet increasing demand, in finding how experiments should be carried out or surveys conducted to obtain reliable results, in determining the order in which operations should be done so as to avoid idle time on machines, in determining the route for a travelling salesman so that he covers all cities in a minimum time, in controlling the traffic density on roads, in determining the correct storage policy for dams, in determining strategies for war, in deter-

mining optimal orbits for space trajectories, in determining how races migrated in history, in determining how blood flows in our veins, in determining whether a two-party or a multi-party system is more stable and even in determining whether a particular poem could have been written by a particular poet. The rate at which the scope of applied mathematics has been widening has been so rapid that we cannot even dare to predict the new directions it is going to take. Truly applied mathematics has realised its freedom and is now advancing more rapidly than ever before.

304. The memoirs in pure mathematics published in the world during a normal year cover several thousands of pages ; mathematical science is enriched year by year by a mass of new results and it spreads and branches out steadily into theories. No mathematician, even were he to devote all his time to this task, would be able to follow all the details of this development. Many mathematicians take up quarters in a corner of the domain of mathematics, which they do not intend to leave, not only do they ignore completely what does not concern their special field, but they are unable to understand the language and the terminology used by colleagues who are working in a corner remote from their own. It is legitimate to ask whether this exuberant proliferation is the external manifestation of a tendency towards a progressive splintering, inherent in the very nature of mathematics, whether the domain of mathematics is not becoming a tower of Babel. In other words do we have today a mathematic or do we have several mathematics ?

—BOURBAKI

305. Today, we believe, however that the internal evolution of mathematical sciences has, in spite of appearance, brought about a closer unity among its different parts so as to create something like a central nucleus that is more coherent than it has ever been. The essential aspect of this evolution

has been the systematic study of the relations existing between different mathematical theories and which has led to what is generally known as the axiomatic method.
—BOURBAKI

306. It is a meaningless truism to say that 'deductive reasoning' is the unifying principle for mathematics. This is equivalent to saying that physics and biology form a single science because both use the experimental method. This method of reasoning is a language suited for mathematics for communication. It is a useful aspect of axiomatic method, but a least interesting one. The real aim is profound intelligibility of mathematics.
—BOURBAKI

307. Just as the experimental method starts from the a priori belief in the permanence of natural laws, so the axiomatic method has its cornerstone in the conviction that mathematics is not a randomly developing concetention of syllogism, nor is it a collection of more or less 'astute' tricks, arrived at by a lucky combination in which purely technical cleverness wins the day. Where the superficial observer sees two or more distinct theories lending unexpected support through the intervention of a mathematician of genius, the axiomatic method teaches us to look for deep underlying reasons for such a discovery, to find the common ideas of these theorems and to bring these ideas forth and to put them in their proper light.
—BOURBAKI

308. Indeed every mathematician knows that a proof has not really been "understood" if one has done nothing more than verify step by step, the correctness of the deductions of which it is composed and has not tried to get a clear insight into the ideas which have led to the construction of this particular chain of deduction in preference to every other one.
—BOURBAKI

309. We cannot overemphasize the fundamental role played in mathematical research by a special intuition (which like all other intuitions can also be

wrong), which is not the popular sense-intuition, but rather a kind of direct divination (ahead of all reasoning). For the research worker who suddenly discovers a structure in the phenomena he is studying, it illuminates with a new light the mathematical landscape in which he is moving about.
—BOURBAKI

310. Mathematics is a study of sets with structures. Mathematics is not a purely mechanical game of isolated formulae, more than ever intuition dominates in the genesis of discoveries. But henceforth it possesses the powerful tools furnished by the theory of the great types of structures; in a single view it sweeps over immense domains, now unified by the axiomatic method, but which were formerly in a completely chaotic state. —BOURBAKI

311. The mathematical structures are not immutable, neither in number, nor in their essential contents. It is quite possible that the future development of mathematics may increase the number of fundamental structures, revealing the fruitfulness of new axioms or of new combinations of axioms. We can look forward to important progress from the invention of structures, by considering the progress which has resulted from known structures. On the other hand, these are by no means finished edifices; it would indeed be very surprising if all the essence had already been extracted from their principles. Thus we can become better aware of the internal life of mathematics, of its unity as well as of its diversity. It is like a big city, whose outlying districts and suburbs encroach incessantly, and in a somewhat chaotic manner on the surrounding country, while the centre is rebuilt in a majestic order, tearing down the old sections with their labyrinths of alleys and projecting towards the periphery new avenues, more direct, broader and more commodious. —BOURBAKI

312. From the axiomatic point of view, mathematics appears to be a storehouse of abstract forms—the

mathematical structures ; and it happens —without our knowing why—that certain aspects of emperical reality fit themselves into these forms, as through a kind of preadaptation. Of course, it cannot be denied that most of these forms had originally a very definite intuitive control ; but it is exactly by throwing out this academic content, that it has been possible to give these forms all the power which they were capable of displaying and to prepare them for new interpretation and for the development of their full power.

—Bourbaki

313. The unity which the axiomatic method gives to mathematics is not the armour of formal logic, the unity of a lifeless skeleton ; it is the nutritive fluid of an organism at the height of its development, the subtle and fertile research instrument to which all the great mathematical thinkers since Gauss have contributed, all those who in the words of Dirichlet have laboured to 'substitute ideas for calculations'.

—Bourbaki

314. For us a civilization without mathematics is unthinkable. Like the parallel postulate, the postulate that mathematics will survive has been stripped of its 'evidence', but while the former is no longer necessary, we would not be able to get on without the latter.

—A. Weil

315. Mathematics is a useless science. By this I mean that it can contribute directly neither to the exploitation of our fellowmen, nor to their extermination.

—G. H. Hardy

316. It is certain that few men of our times are as completely free as the mathematicians in the exercise of their intellectual activity. Let others besiege the offices of the mighty in the hope of getting the expensive apparatus, without which no Nobel prize comes their way. Pencil and paper is all that the mathematician needs, he can even sometimes get along without these. Neither are there Nobel prizes

to tempt him away from slowly maturing work, towards a brilliant but ephemeral result.
—A. WEIL

317. We have learned to trace our entire science (mathematics) back to a single source, constituted by a few signs and by a few rules for their use ; this is unquestionably an unassailable stronghold into which we are always free to retire in case of uncertainty or external danger. Only a few backward spirits still maintain the position that mathematician must for ever draw on his 'intuition'. If certain branches of mathematics have not yet been axiomitised, this is simply because there has not yet been time to do it.
—A. WEIL

318. In recent times, mathematics has demonstrated its vitality by passing through one of these periods of growing pains, to which it has been accustomed for a long time and which are designated by the strange name of "foundation crisis". It has come through it, not only without damage, but with great gain.
—A. WEIL

319. If logic is the hygiene of the mathematician, it is not his source of food ; the great problems furnish the daily bread on which he thrives. —A. WEIL

320. A branch of science is full of life as long as it offers an abundance of problems, a lack of problems is a sign of death. Mathematics is always full of life as there is always an abundance of problems.
—D. HILBERT

321. We neither can, nor want to lay a route for the future development of our science ; this will be a futile task, indeed it would be a ridiculous enterprise ; for the great mathematicians of the future, like those of the past, will flee from the beaten path. They will solve the great problems through unexpected connections which our imagination will not have succeeded in discovering and by looking at them in a new light. —A. WEIL

322. Mathematics is an organism for whose vital strength the indossuble union of its parts is a necessary condition.

—D. Hilbert

323. When a branch of mathematics ceases to interest any but the specialists, it is very near to its death or at any rate close to its paralysis, from which it can be rescued by being plunged back into the vivifying sources of the science.

—A. Weil

324. Does this mean that mathematics is becoming a science for erudites and that it will no longer be possible to do creative work in mathematics until one has grown grey in harness and exhausted from burning of midnight oil for many years in the company of dusty tomes ? This would be a sign of its decline ; for be it strength or weakness, mathematics is not a science that prospers on details, painfully collected in the course of a long career, or patient reading, on observation or on filling of cards arranged one by one so as to form a bundle from which an idea will ultimately come forth. Perhaps it is more true in mathematics than in any other branch of knowledge that the idea comes forth in full armour from the brain of the creator. Moreover, mathematical talent usually shows itself at an early age ; and the workers of the second rank play a smaller role in it than elsewhere. There are examples to show that in mathematics, an old person can do useful work, even inspired work ; but they are rare and each case fills us with wonder and admiration. Therefore if mathematics is to continue to exist in the way in which it has manifested itself to its votaries until now, the technical details in it must be superficial or of temporary character ; in the future, as in the past, the great ideas must be simplifying ideas. The creator must be one who clarifies, for himself and for others, the most complicated formulae and concepts.

—A. Weil

325. The mathematician believes that he will be able to slake his thirst at the very sources of knowledge,

convinced as he is that they will always continue to pour forth, pure and abundant; while others have to have recourse to the muddy streams of sordic reality. If he be reproached with for the haughtiness of his attitude, if he be asked why he persists on the high glaciers where no one but his kind can follow him, he answers with Jacobi, "For the honour of the human spirit." —A. Weil

326. The highly abstract character of mathematics might lead one to suppose that, whatever its fascination for a somewhat eccentric class of human beings, it cannot be of much value at least in its higher flights – in solving concrete problems. There can be no greater mistake. In fact the highly abstract study appear to have a range of applicability second to none, and the reason for this is very illuminating. The higher and further away from the terrafirma of the so called external world that man climbs up, the more extensive is his vision and the more effective is his ability to function thereon. It is because mathematics has freed itself from the study of any particular category of sense experiences in sharp contrast to the other sciences and functions at a higher level of abstraction, that it is able to provide an armoury of weapons suitable for other sciences.
—A. Narasinga Rao

327. Pure mathematician is a Brahma creating thought forms of entracing beauty at a level higher than that of sense experience and using a language which evolved at a later stage of evolution than ordinary speech. —A. Narasinga Rao

328. It seems to me that the main problem of mathematical education consists in the transmission to the young of the intellectual excitement implicit in this endeavour and inculcation in them of an appreciation of its significance. No one who has not experienced the thrill of mathematical discovery or gained a perspective of the vast expanse of

mathematics at some angle, can possibly inspire the young to become good mathematicians.

—K. CHANDRASEKHARAN

329. While the cultivation of mathematics, either in the sense of making new discoveries, or of assimilating known theories, is an exciting form of intellectual activity, it should not be forgotten that mathematics has amply demonstrated its utility and power in its interaction with other sciences, be they physical, social or biological. In fact the dominant characteristic of modern scientific thought may truly be described as mathematical. We cannot expect first rate technology to grow up besides third rate mathematics. The attainment of a respectable level of mathematics culture should therefore be an immediate goal.

—K. CHANDRASEKHARAN

330. All teachers should take their tickets for adventure ; and their starting point should be the study of modern mathematics. Modern mathematics is an extraordinary new world, it has such a beauty that love will come, and then they will communicate that spark of love to their pupils. To teach mathematics without knowing modern mathematics would be to act like a museum curator who has in his museum some old painting and who refuses to believe that there exists a modern school of painting. A teacher should not be a museum custodian, he should become himself a creator and to find out and arouse creativity in his pupils.

—GUSTAVA CHOQUET

331. The fundamental structures of mathematics can be compared to those machines which can manufacture a single kind of object, but are able to manufacture a large number of copies within a short time and moreover manufacture each of them with a high degree of perfection. The time we spend in studying these big machines is well spent, for the theorems we get do not concern any more single mathematical entity but a large class of functions, of

spaces, and their proofs have character of great elegance, simplicity and economy.

—Gustava Choquet

332. Geometry, as a logical system, is a means and even the most powerful means—to make children feel the strength of the human spirit, that is of their own spirit.

—H. Freudenthal

333. Mathematics is a language to be used and enjoyed ; few would quarrel with the first part of the description, the second would arouse more criticism, yet surely it is as important as the first.

—T.A.A. Broadbent

334. Freedom in mathematics is the freedom of the creative artist to create his own patterns. It is akin to the freedom which the painter or the musician or the sculptor or the architect needs for his full self expression. Just like any one of them, the mathematician wants to be free to do his creative researches, guided only by his innate sense of beauty, symmetry, consistency, harmony and aesthetics. He does not need brushes and paints or musical instruments or even stone and chisel, since he works in the world of pure thought. His only instruments are pencil and paper and with these he weaves patterns of symbols which are in reality patterns of ideas.

*335. It is no exaggeration to say that not more than one percent of the structures which the mathematicians have created have been responsible for the revolutionary advances in science and technology. Take away these one percent structures and the whole fabric of science and technology will come tumbling down.

*336. The attitude of society to the creation of new mathematics is similar to that of the royal patrons of old to the creation of painting and music. The king used to permit his painter to paint as he

liked throughout the year provided once a while he would paint the royal portrait. He would permit his poet to compose the poems he likes provided once a year he composed a poem in his praise. Similarly society today allows mathematicians most of the time to create their mathematics according to their criteria of consistency and beauty provided sometimes they create structures useful to science and technology. The society realises that the small portion of useful mathematical structures are so valuable that they many times overpay for the investment in the creation of other structures.

337. The object of mathematical rigour is to sanction and legitimatize the conquests of intuition, and there never was any other object for it.

—HADAMARD

338. To introduce new concepts without a sufficient background of concrete facts, to introduce unifying concepts where there is no experience to unify, or to harp on the introduced concepts without concrete applications which would challenge the students, is worse than useless : premature formalisation may lead to sterility, premature introduction of abstractions meets resistance especially from critical minds who before accepting an abstraction, wish to know why it is relevant and how it could be used.

339. Mathematical thinking is not just deductive reasoning ; it does not consist merely in formal proofs. The mental processes which suggest what to prove and how to prove it are as much a part of mathematical thinking as the proof that eventually results from them. Extracting the appropriate concept from a concrete situation, generalising from observed cases, inductive arguments, arguments by analogy and intuitive grounds for an emerging conjecture are mathematical modes of thinking. Indeed, without some experience with

such "informal" thought processes the student cannot understand the true role of formal rigorous proof. There are several levels of rigour. The student should learn to appreciate, to find and to criticize proofs on the level corresponding to his experience and background. If pushed prematurely to a too formal level, he may get discouraged and disgusted.

340. It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilable when it is in the nascent state. —MAXWELL

341. In its cultural significance as well as in its practical use, mathematics is linked to other sciences and the other sciences are linked to mathematics, which is their language and their instrument. Mathematics, separated from the other sciences loses one of its most important sources of interest and motivation.

342. "What use is the calculus ?"

"What use is the baby ? Wait and see."
—NEWTON

343. The duty of every mathematician is to learn a little new mathematics, to teach a little new mathematics and to create a little new mathematics.
—J.L. KELLEY

344. The only way to learn mathematics is to recreate it for oneself. —J.L. KELLEY

345. The objects of mathematical study are mental constructs. In order to understand these one must study, meditate, think and work hard.
—SHANTI NARAYAN

346. Mathematicians are generally intercsted either in proving old theorems by new methods or in proving new theorems about old mathematics.

347. In this era of expanding science, modern mathematics is like a jet plane streaking across the sky to

an adventurous destination. But to the average by stander who watches the jet-stream, even its take off point is a mysterious hidden secret.

—I. ADLER

348. Mathematics is like a chest of tools, before studying the tools in detail, a good workman should know the object of each, when it is used and what it is used for.

—W W. SAWYER

349. You can claim to be a mathematician, if and only if you feel that you will be able to solve a puzzle that neither you, nor anyone else has studied before. That is the test of reasoning.

—W.W. SAWYER

350. Mathematics and music, the most sharply contrasted fields of scientific activity and yet related, supporting each other, as if to show forth the secret connection which ties together all the activities of the mind and which leads to the surmise that the manifestations of the artist's genius are the unconscious expression of a mysteriously acting rationality.

— H. HELMHOLTZ

351. Mathematical theories do not try to find out the true nature of things, that would be an unreasonable aim for them. Their only purpose is to coordinate the physical laws we find from experience but could not even state without the aid of mathematics.

—A. POINCARE

352. In a sense our subject matter is old. However the axiomatic development which we have adopted here is comparatively new. A beginner may find our account at times uncomfortably abstract since we do not study one particular system (*e.g.* the system of real numbers). At any rate it will be obvious that much time is saved and a clearer insight is eventually achieved by the present method.

—N. JACOBSON

353. I have taught mathematics to almost every kind of student. In my experience there is hardly any man

who may not become a discoverer, an advancer of knowledge and the earlier the age at which you give him chances of exercising his individuality, the better.
—JOHN PERRY

354. Mathematics in earnest should be fun ; mathematics in fun may be earnest.
—N. A. COURT

355. The reasons for declaring oneself satisfied with a reasoning are of a psychological nature ; in mathematics as in everything else.
—H. LEBESGUE

356. In the body of mathematical culture, there is action and reaction among the various elements. Concepts react upon concepts ; ideas mix, fuse, form new syntheses. When this process of interaction and development reaches a certain point, new syntheses are formed of themselves. These are, to be sure, real events and hence have their location in time and space. The places are, of course, the brains of man. Since the cultural process has been going on rather uniformly over a wide area, the new synthesis may take place in a number of brains at the same time.
—L.A. WHITE

357. Axiomatics cannot banish ingenuity from mathematics.
—BELLMAN

358. Oliver Heaviside stated, "I know mathematical processes that I have used with success for a very long time, of which neither I nor anyone else understands the scholastic logic. I have grown into them and so understand them that way." He almost scorned not only mathematical rigour ("Shall I refuse my dinner because I do not fully understand the process of digestion ?") but, it sometimes appeared, even mathematicians (even Cambridge mathematicians) deserve justice. Yet inspite of his lack of rigour, he contributed as much to mathematics as any other mathematician. on account of his intuition.

359. The ultimate aim of workers in the foundations of algebraic geometries is to erect an aesthetically pleasing structure, free from logical faults, on which the many ornaments of Italian geometry can be tastefully displayed.
—D. Pedoe

360. Most of the essays reprinted in 'Models of Man' make considerable use of mathematics. The choice of mathematics over other languages was based largely on utilitarian rather than aesthetic grounds. I found mathematics indispensable in enabling me to discuss questions about human behaviour in groups and organisations, clearly and precisely. In several cases, the mathematical methods led me to the solutions of problems that had baffled me and others for a decade or more.
—Simon

361. Mathematical analysis is as extensive as nature itself; it defines all perceptible relations, its chief attribute is clearness; it has no marks to express confused notions. It brings together phenomena the most diverse, and discovers the hidden analysis which unites them. It seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfections of the senses.
—Fourier

362. The "moment of discovery" of the great medical breakthroughs has often been memorialised in the pictorial arts. In the future the physician or biologist will still be there at the moment of triumph. But look at the picture closely and you might see a computer and a mathematician standing over the corner.
—Random News

363. In mathematics, as in any other scientific research, we find two tendencies present. On the one hand, the tendency towards abstraction seeks to crystallise the logical relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency towards intuitive

understanding fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete among their relations. —D. HILBERT

364. In geometry, the abstract tendency has led to the magnificent systematic theories of algebraic geometry, of Reimannian geometry and of topology; these theories make extensive use of abstract reasoning and symbolic calculus in the sense of algebra. Notwithstanding this, it is still as true today as it ever was that intuitive understanding plays a major role in geometry. And such concrete intuition is of great value not only for the research worker, but also for any one who wishes to study and appreciate the results of research in geometry. —D. HILBERT

365. One of the self-imposed, and sometimes unappreciated, tasks of the mathematician is that of providing a choice of a firm basis for quantitative description of natural processes. Among the several motivations for these Herculean labours, the fact that they are interesting is certainly paramount. The physical world has been and continues to be the primary source of intriguing and significant mathematical problem. Although one would think that the armchair philosopher with his ability to conjure up countless infinities of universes could easily create arbitrarily many fascinating fictitious worlds, historically this has not been the case. The mathematician, from all appearances, needs the constant infusion of ideas from the outside. Without these stimuli, the pure bread of axiomatics, as pure breads are wont to do, becomes sterile and decadent. —R. BELLMAN

366. Another motivation for the study of mathematics is pragmatic. Predicted results, derived from mathematical models of physical phenomena, can be compared with experimental data obtained from a study of the actual physical phenomena, and thus

used to test the validity of the fundamental assumptions of a physical theory. Mathematics can therefore, if not capable of constructing the actual universe by the critique of pure reason, at least plays an essential role demonstrating what hypotheses should not be put forth. —R. Bellman

367. Mathematics is too intimately associated with science to be explained away as a mere game. Science is serious work serving social ends. To isolate mathematics from social conditions which bring mathematicians into existence is to do violence to history. —Jagjit Singh

368. It is taken as a modern axiom that the use of mathematics in any field clarifies problem formulation and contributes problem solution and enhances the reputation of the field. We see, consistent with this assumption, a great deal of mathematical activity in economic and industrial research, in the field of psychological research, and even, by way of game theory, in politics. —R. Bellman

369. It is axiomatic, however, that it is worthwhile to examine the possibility of the application of mathematical techniques to any scientific field. We know that a great deal has been gained from this in the past and the introduction of the digital computer greatly increase the promise of the future.

—R. Bellman

370. It must be remembered that the mathematician is not interested, and rightly so, solely in being useful. His fundamental objective is to do creative mathematics. Generally speaking, he is pleased to see some of his results used by the scientific community, but this is a minor criterion for pleasure compared to his principal goal. —R. Bellman

371. Since applications of mathematical theories to significant scientific problems are never routine and straightforward, we obtain in this fashion a valuable feedback to the field of mathematics itself of

new ideas and new techniques which can be applied to still other new scientific problems, and so the research process continues, one field stimulating the other. —R. Bellman

372. Who has taught us the true analogies, the profound analogies which the eyes do not see, but which reason can divine ? It is the mathematical mind, which scorns content and clings to pure form.

—H. Poincare

373. To many scientists, the mathematician when regarded as at all useful is looked upon as a walking table of integrals He is the court of last resort when the engineer or physicist is faced by an intractable differential equation. The situation however is changing very rapidly because of the high level of mathematical training of engineers and physicists, most of whom can recognize a tame equation as quickly as a professional mathematician, because of the availability of digital computers which make special analytic solutions of little importance and because of the training of mathematicians more and more in abstract terms and less and less in problem solving. Thanks to the influence of the Bourbaki school, there is almost as much chance of finding ingenius solutions of mathematical problems in papers of engineers and physicists as in mathematical papers.

—R. Bellman

374. The narrow, isolated specialist in mathematics does without doubt make a contribution, but he is not to be compared with the great mathematicians of history who have always understood and used the emperical sources of mathematical inspiration. Not every application is of genuine mathematical interest, but this does not justify the pure mathematician's tendency to make a virtue of a lack of interest in mathematical problems arising in other sciences. —La Salle

375. Historically, mathematics has always possessed a paradoxial duality. On the one hand mathematics

reflects upon itself and is the most original creation of the human mind ; yet there has always existed a close relation between mathematics and those emperical sciences that have risen above the level of pure description. Regardless of its purity, mathematics has the habit of proving to be useful.

—La Salle

376. The danger is that a mathematical field isolated too long from its emperical source may undergo an abstract inbreeding that finally results in a sort of killing cancerous growth. The discipline splits into a multitude of subdisciplines and tends to become a disorganised mass of detail and complexity. The orderly growth of mathematics is inspired and directed by the realities of the emperical sciences, but it remains true that the strength and power of mathematics is its own creation. We should force no dichotomy on mathematics. —La Salle

377. One of the great advantages of the mathematical approach to problem solving lies in the fact that analytic techniques developed to treat specific questions in one scientific area can readily be applied to any area in which analogous functional relations hold. It follows that there is a constant interplay between the general and the particular. General methods come out of the investigation of particular problems, and particular problems are resolved by means of powerful general methods.

—R. Bellman

378. Considering the relatively small dent that mathematical analysis has made in various fields of engineering, we cannot be too optimistic about what it can do in the enormously more complex fields of biology and medicine where we barely know how to formulate problems, much less solve them. The skilled mathematician however possesses a vast collection of mathematical formulations of physical processes with associated computational algorithms. This skill combined with the experience and intuition of a biomedical research man can

result in significant mathematical models of biological processes. One of the functions of the mathematician interested in the areas of biology and medicine is to convince his brethren that there are significant and intriguing mathematical questions in these new fields and by example to show the biologist and medical researcher that he can contribute to their problems. —R. BELLMAN

379. The application of mathematical techniques to biology was mentioned in the literature as early as 1922. In the 1930's there were perhaps a dozen papers on the subject ; in the 40's there may have been about a hundred, and by 1960 so many papers were written that no single bibliography was able to encompass them all. Though this is a dynamic, promising field, it is too early to hail the mathematician and the computer as the key that will unlock all the mysteries of the human body. The difficulties in making mathematical models are formidable. If the assumptions made in constructing the model are too broad, only general statements can be made ; if they are highly simplified in the model so that it will yield a solution, the mathematician runs the risk of losing biological significance.

—RANDOM NEWS

380. It is essential to examine the quiet revolution which has taken place in mathematics in our own times and to appraise its now enormous potentialities for bringing fundamental advances in every domain where reason and scientific thinking have roles to play. —M. H. STONE

381. What has to be grasped by all who are interested in education is that our conception of the nature of mathematics has been revolutionised, our technical knowledge of the subject vastly enriched, and our dependence upon it for scientific and technological progress enormously increased. Indeed it is becoming clearer and clearer everyday that mathematics has to be regarded as the corner-stone of all scienti-

fic thinking and hence of the intricately articulated technological society we are busily engaged in building. We can foresee a time in the not very distant future when a complete identification of science, logic and mathematics will be achieved.

—M. H. STONE

382. Mathematics is now seen to have no necessary connections with the physical world beyond the vague and mystifying one implicit in the statement that thinking takes place in the brain. The discovery that this is so may be said without exaggeration to mark one of the most significant intellectual advances in the history of mathematics.

—M. H. STONE

383. It has now been possible to determine quite precisely the connection between mathematics and logic, and to define the scope of mathematics in terms so broad and so simple that they appear to provide a nearly final answer to the question, "What is Mathematics ?". It has been possible to see how the different branches of mathematics are united into one imposing whole and to reinforce the technical bonds which tie them together within the framework of a general definition. It has been possible to win our way to a new point of vantage from which we are able to appreciate the remarkable technical advances achieved in mathematics over the last sixty years as merely the prelude to greater accomplishments in the future. —M. H. STONE

384. When we compare the mathematics of today with mathematics as it was at the close of the nineteenth century, we may well be amazed to note how rapidly our mathematical knowledge has grown in quantity and in complexity, but we should not also fail to observe how closely this development has been involved with an emphasis upon abstraction and an increasing concern with the perception and analysis of broad mathematical patterns. —M. H. STONE

385. This new orientation made possible only by the divorce of mathematics from its applications, has been the true source of tremendous vitality and growth during the present century. We realise too that this trend towards abstraction must inevitably continue, reinfoined by the successes which are already to be credited to it. —M. H. STONE

386. In directing their attention more and more to the discernment and study of abstract patterns, mathematicians have become increasingly aware of the fundamental antithesis between the structural aspects of mathematics and the strictly manipulative aspect which so often appears to have paramount importance for the applications and so often is the principal preoccupation of the mathematics teacher.
—M. H. STONE

387. A modern mathematician would prefer the positive characteristic of his subject as the study of general abstract systems, each one of which is an edifice built of specified abstract elements and structured by the presence of arbitrary but unambiguously specified relations among them. He would mean by the study of such mathematical systems not only the examination of the intrinsic properties of individual systems but also the comparison of the structures of different systems. —M. H. STONE

388. The mathematician would maintain that neither these systems nor the means provided by logic for studying their structural properties have any direct, immediate or necessary connection with the physical world. At the same time, he would recognise that such mathematical systems can often usefully serve as models for portions of reality, thus providing the basis for a theoretical mathematical analysis of relations observed in the phenomenal world. He would also acknowledge that this kind of accidental and to a certain extent arbitrary connection between some part of reality and certain mathematical system has often led to the discovery of abstract features of

the latter which would eventually be made the subject of abstract mathematical proof.

—M. H. STONE

389. The fusion between mathematics and logic reinforces the conclusion that mathematics is a completely abstract formal discipline and raises interesting question as to how much can be accomplished by the mechanical manipulation of symbolic systems and how much depends upon direct or intuitive insights into their structural patterns. —M. H. STONE

390. It is clear that mathematics may be likened to a game—or rather to an infinite variety of games—in which the pieces and moves are intrinsically meaningless and absorbing interest lies in perceiving and utilizing the patterns allowed under the rules.

—M. H. STONE

391. Even though we have no warrant for supposing that the mathematical investigations of the future will necessarily use techniques or disclose phenomena generally like those known at the present time, we may still expect the new mathematics, however diversified, to be embraced in the unity which is so broad and admits so high a degree of abstraction that the limits it sets to mathematics can hardly be pushed back any further. —M. H. STONE

392. It may seem to be a stark paradox that, just when mathematics has been brought close to the ultimate in abstractness, its applications have begun to multiply and proliferate in an extraordinary fashion. There is no doubt that one of the most exciting features of intellectual life in the twentieth century is the penetration of mathematics into ever widening circles of scientific disciplines, not only the natural sciences but also those devoted to the study of human behaviour. —M. H. STONE

393. A whole new world of thought and understanding opens out before us to which mathematics alone is the key. Far from being paradoxial, however, this conjunction of two apparently opposite trends

in the development of mathematics may rightly be viewed as the sign of an essential truth about mathematics itself. For it is only to the extent that mathematics is freed from the bonds which have attached it in the past to particular aspects of reality that it can become the extremely flexible and powerful instrument we need to break up paths into areas now beyond our ken. —M. H. STONE

394. In the last three or four years the number of active mathematicians as measured in terms of publication, seems to have doubled. There is another strong evidence that mathematics is experiencing a literally explosive growth. More significant than anything else is the increasing rate at which important new ideas and techniques are being introduced into mathematics. —M. H. STONE

395. The result of Church and Turing suggests that logic and methematics cannot be reduced entirely to a routine programme of mechanical manipulation with symbols ; that there may be some things possible for the human mind which any particular machines cannot do. Thus the antithesis between manipulating symbols and perceiving patterns may perhaps be related to the logical phenomenon discovered by Church and Turing, and may be genuinely rooted in the distinction between mind and machine. —M. H. STONE

396. A difficult and challenging problem is that of coordinating the teaching of pure mathematics, with the teaching of widely different fields in which mathematics is being applied. The problem already critical is made all the more pressing because of the abstract tendencies apparent in present day mathematics and its involvement with the ever more sharply defined antithesis between the manipulative and structural aspects of mathematics.
—M. H. STONE

397. Since abstraction and discernment of patterns are demonstrably playing more important roles in the

scientific study of nature, the mathematicians desire to emphasize the abstract and the structural aspects of mathematics, both because of their intrinstic interest and because of the firmer grasps they give of concrete situations, and manipulative techniques may today begin to be more sympathetically received. It is important however that the mathematician should not neglect the manipulative aspects of his subject, nor insist that his own concern for generality and precision should at all times and under all circumstances be shared by those who engage in making applications of mathematics.

—M. H. Stone

398. Applied mathematics is a branch of science which seeks knowledge and understanding of the external physical universe through the use of mathematical methods and scientific inference. The ultimate goal of the efforts of the applied mathematician lies in the creation of ideas, and that bears repeating in the creation of ideas, concepts and methods that are of basic and general applicability to the subject in question, be it elasticity, magnetohydrodynamics, geophysics, biochemistry, information theory or even economics. The ideal applied mathematician is a truly versatile scientist—a specialist in mathematics—with broad and active interests in many scientific areas.

—H.P. Greenspan

399. There are three principal facts in the applied mathematician's approach to any particular problem, each of equal importance. Firstly he must formulate the physical problem in mathematical symbolism, secondly, he must solve the mathematical problem and thirdly, he must discuss, interpret and evaluate the results of his analysis. —H.P. Greenspan

400. Mathematics in combination with scientific inference provides the most powerful tool for the advancement of science yet devised. It is the applied mathematician who is best trained to use this knowledge.

—H. P. Greenspan

401. Applied mathematics cannot and should not wait until mathematical knowledge has improved sufficiently for him to turn out a polished piece of work. His real business is primarily with the understanding of physical phenomena and if he can contribute to this end, he should proceed, doing always the best he can with the mathematics and struggling always to improve it. —S. GOLDSTEIN

402. The applied mathematician is generally the more knowledgable, as compared with a theoretical physicist concerning mathematical theories, methods and techniques and is certainly concerned with a broader spectrum of problems. Any and all things that excite his interest are amenable to mathematical formulation and analysis are within his province and this ranges from studies of liquid helium, kinetic theory and meteorology, to analysis of circulation of blood, the swimming of fish and automobile traffic control. —H.P. GREENSPAN

403. The aim of applied mathematics is to simplify the description without losing the physical core of the phenomenon. Experience is necessary to develop the flair for making just the right approximation in the always complicated physical situation. The talent is not easily come by. Thinking in physical terms must be started early in the training of the applied mathematician. —H.P. GREENSPAN

404. The applied mathematician must, of course, be extremely proficient and creative in the use of mathematical methods and techniques. Much of modern mathematics arose from the need to solve physical problems. In fact most of the great mathematicians of the last century were both pure and applied, attesting to the stimulation of one upon the other. Physics has probably given more to pure mathematics in the way of stimulation than it has received. —H. P. GREESPAN

405. The fact that much of modern mathematics is now so far removed from physical reality leads me to

question any statement which suggests that "it is in pure mathematical research that future practical uses will be based". —H.P. GREENSPAN

406. The applied mathematician can find the solution to any difficulty, whereas the pure mathematician can find the difficulty to any solution.

407. Since the primary aim of the applied mathematician is directed to the understanding of physical phenomena, he must often use approximate methods or invoke physical reasoning to achieve a simplification often without being able to estimate the errors involved. All of these means are employed because most current research involves highly non-linear problems about which very little is as yet known. Although his treatment is all the time responsible and disciplined, the applied mathematician is not a deductive logician interested solely in the beauty of form and the power of abstraction. In other words he is not a pure mathematician. He must, however, have enough background to distinguish between clear demonstration, plausible arguments, and hopeful speculation —all of which are used and used often. Rigour is of course always desirable but not at the price of sterility.
—H. P. GREENSPAN

408. Our best scientific tools go unused because they require a mathematical sophistication not always possessed even by the foremost specialists in subjects like astrophysics, meteorology, oceanography, economics and social sciences. How much there is to be done, to be understood, appreciated and incorporated into the body of knowledge ? We need not look too far to find a 'New Frontier'. It is no wonder that the Russians consider applied mathematics important and emphasize it strongly.
—H. P. GREENSPAN

409. I maintain that there should and must be room in mathematics proper for both the pure and the applied. The present situation is grossly unbalanced and

represents a potential source of difficulty. A re-adjustment and reappraisal is vitally necessary. Mathematics curriculum must begin to account for the fact that not everyone desires to be a pure mathematician nor is such training the only or best possible or even the most desirable. Finally, I ask you to ask yourselves, "Is the domination of American mathematics by the pure mathematicians warranted, desirable, desired, necessary and is it really in the national interest ?"

—H. P. GREENSPAN

410. Specialization leads, on the one extreme, to the pure mathematician who is concentrating on just one small point in some abstract point set and on the other to the applied mathematician who is concentrating on the intricacies of a large intelligent inefficient and probably unnecessary computer programme. Each scorns and is scorned by the other. It is my claim that the points of view of both caricatured by these two extremes belong together. They are compatible with each other and for both the rigour and realism of mathematics we need each. Alone they can be sterile. I propose a marriage of the two points of view.

—T. E. HULL

411. Mathematics has retained its place among the chief subjects of education for well over two thousand years but this does not mean that it is surely immortal. It has stood firmly because it has stood on two legs. First, it is supported by its innate beauty and austere elegance. Second, it is supported by its usefulness to scientists and technicians of all kind. If we try to make it stand exclusively on its usefulness, it becomes a mere tool for the use of the non-mathematicians and degenerates into dullness and eventually into uselessness. If we try to make it stand exclusively on its aesthetic virtues, we not only make it useless to other sciences, but reject the stimulus that it can receive from them. So it is desirable that among the research workers in mathematics, there should always be some who are in-

terested in applications. Likewise, the scholars and the teachers should not ignore the many uses of mathematics. Right now this places quite a demand on the scholars and teachers, for mathematics has entered new fields sometimes rather in unexpected ways. —E. J. MACSHANE

412. The normal activities of a mathematician should include a mixture of three ingredients. One is teaching, another is research, and the third which is all too often left unmentioned, is scholarship. Any one of these three may be omitted, but in my opinion not without loss. —E. J. MACSHANE

413. Communication between the teacher and the scholar of mathematics is so important that I can imagine only one satisfactory arrangement : the two must wear the same skin. The man who is neither researcher nor scholar, and nevertheless is listed on some payroll as a teacher of mathematics is falsely listed. At best he is a transmitter of information given him in the more or less distant past, at worst he is a drill master for the problems in some third-rate text book. The man who is a research specialist and teacher, without being a scholar, is sometimes a teacher only by necessity, giving all his enthusiasm to his research. Sometimes he is an enthusiastic teacher of advanced students, spreading forth the pleasures of specialisation in his own field. In this case he may produce students with great enthusiasm but narrow views, whose broader education must be left to other more scholarly teachers or in the hands of the gods. —E. J. MACSHANE

414. Quite another matter is the communication of thought between the research worker on the one hand and the teacher-scholar on the other. Here is the domain of usefulness of the expository articles of which the supply is scant. A chief reason for this scantiness is that since a research paper, even on the narrowest and most special of topics, is ordinarily looked on with more reverence

than even an excellent expository paper, it is natural and human that a mathematician should be inclined to spend all his available working time on research. The form of scholarship that supports the writing of a good expository paper comes from having read, marked, learned, and inwardly digested many articles in some field. Usually this reading was prompted by an interest in some research problem, but this is not a necessary condition. However, the mere intensive reading of many papers is not a sufficient condition either. One can be so imbued with interest in the special problem that everything is mapped on a sort of polar coordinates with one special problem at the origin and interest inversely proportional to r^2. Alternatively, one can read something new to find what it is in itself and how it relates to previous knowledge. Such reading, with thoughtful rumination, is a natural source of scholarship, of good teaching and in particular of good expository writing. —E. J. MACSHANE

415. Every mathematician owes a debt to mathematics that he should repay by one hard job of scholarly writing. —L. E. DICKSON

416. If any man wishes to consider himself a teacher and a scholar of mathematics, it is his clear duty and it should be his pleasant duty, to add continually to his knowledge and in this he should be greatly helped by expository articles of the type appropriate to him. The teacher who neglects this is doing himself a great harm. Any teacher of mathematics who thus decides that mathematics belongs to the Valley of Dry Bones must inevitably convey the same impression to his students. The best of books is inferior to a human being as a means of conveying enthusiasm for and pleasure in a field of study and if the teacher fails to show that the subject is alive and moving and fascinating, he fails in just that respect in which the responsibility is most peculiarly his own. —E. J. MACSHANE

417. Applied mathematics was not long ago regarded as the tedious solution of specific problems by known devices and often without logical justification. Even today the mathematical reasoning in the quantum theory of fields or in nuclear physics is apt to shock a mathematician trained in rigour. There are two ways of reacting to such a situation. One is to look on the physical theory as ludicurous and refuse to sully ones hands with it. The other is to observe that the illogical theory has yielded useful results, and is thus probably a sort of a parody of a rigorous and coherent theory. This implies a challenge to find that rigorous theory. It is in such situations as this that the applications have benefitted mathematics, by calling for new devices and new combinations of old devices to handle a problem which has presented itself not artificially but irrepressibly and clothe with its own importance.

—E J. MACSHANE

418. I cannot look on the proliferation of mathematics as being in all circumstances as unqualified good. Each mathematical discipline needs to draw on the others, yet it is impossible for even the best of us to keep abreast of the research in more than a small part of the field. We are separating off into small groups of specialists with little intercommunication.

—E. J. MACSHANE

419. An addition to mathematical knowledge has significance as a part of the body of all mathematical knowledge, and its place in that body should be clearly indicated. If the author of a research paper has not bothered to establish the setting of his paper and convey some of the motive that impelled him, he has driven off potential readers and made everything harder for those who stay with him. I am convinced that it is the duty of editors to demand, not to forbid, the writing of introductory paragraphs, to provide motivation and background. This need not be long, even a little can be disproportionately helpful.

—E J. MACSHANE

420. The present day development of physics and technology make numerous demands on mathematics which only mathematicians capable of creating new ideas can satisfy. Also for the purposes of instruction, persons are required who view their subject as a living and developing discipline and not as a conclusively formed and ossified system of knowledge. This essential idea must pervade all instruction from its first stages to its conclusion. What demands life will present to him, what mathematics he will have to use, it is impossible to foresee along most general lines. Therefore even in the first school years, it is necessary to develop in children flexibility of the intellect, to inculcate the thought that in the future it will be necessary to add many more forms of knowledge to the scientific foundation that is laid in the school and among these, mathematical ones.

—B. V. GNEDENKO

421. From the teacher of mathematics is demanded enthusiasm for his subject and the conviction that his subject is one of the most important affairs of the nation. From the teacher it is demanded that he implants in the students a love for mathematics and a conviction of the creative power of his pupils and that he describes in general outline before their intellectual gazes, the impressive picture of the uninterrupted development of mathematics with its limitless connection with technology, the natural sciences and other manifestation of human activity.

—B. V. GNEDENKO

422. Aerodynamic science is an excellent example of cooperation between 'men of mathematics' and 'creative engineers'. Mathematical theories from the happy hunting grounds of pure mathematicians are found suitable to describe the air flow produced by aircraft with such excellence accuracy that they could be applied directly to airplane design.

—VON KARMAN

423. The position of the mathematician in our present day technology has not been uniformly established.

Perhaps it can never be. Nevertheless many people today are faced with the question : 'Should the mathematical aspects of this study be carried out by an engineer or scientist with a flair for mathematics or by a mathematician with a flair for the material under study ?' In idler days this question would be interpreted as a bid for jurisdiction, but with work as abundant as it is, the question can only bear upon the efficiency of the utilization of our scientific manpower.

—R. E. GASKELL

424. Custom seems to require that a mathematician spend a century in the grave before a public display of his portrait is permitted. —R. E. GASKELL

425. It is quite true that the usefulness of mathematics has been increased enormously by the availability of high speed computation, but it is a mistake to go further and assume that high-speed computers have replaced or will ever replace mathematics or mathematicians. —R. E. GASKELL

426. Mathematics has been called the queen of the sciences and it has also been referred to as the handmaiden of the sciences. Thus while there is some doubt about the social position of mathematics, at least its sex is definitely known.

—J. D. WILLIAMS

427. Mathematics is thought moving in the sphere of complete abstraction from any particular instance of what it is talking about. —A. N. WHITEHEAD

428. Russel's definition of mathematics as the science in which we do not know what we are talking about, nor whether what we say is true or not, should enjoy almost universal acceptance. The mathematician accepts it, because in all seriousness, there is an element of truth in it, and by the time he explains its weakness, his listeners will agree that it must be precisely correct. The non-mathematician accepts this definition readily—almost eagerly—

because it explains so much of what he has heard and read.
—R.E. GASKELL

429. For the good of our technology, it is imperative that the position of mathematician be established as basic. They should be used continuously and regularly and not just to provide occasional flashes of genius when all else fails. Until this happy day dawns much of his best work in which his ideas and methods provoke an evolution in the designs and decisions of his clients, will be unrecognised and unrewarded. We can speed the coming of this day, if we develop more mathematicians whose thought is problem-centred rather than method centred and whose training is broad enough so that they can give sound advice in a wide variety of problem situations.
—R. E. GASKELL

430. It is the duty and privilege of the mathematicians to create the right type of intellectual climate in which science and technology can flourish.
—P. K. KELKAR

431. Computers tremendously extend the range of mathematical testing, experimentation and inductive motivation into ever expanding areas of the mathematical unknown. They certainly tend to extend the cosmopolitan interest of mathematicians. However they can never eliminate the need for problem-solving through human ingenuity and intelligence.
—R. BELLMAN

432. That gifted young people must be given the fullest opportunity to taste pure mathematics pleasures and be seduced by its charms is a first requirement of any programme of mathematical education. But the times demand that additional group of young mathematical scholars be trained who will seek their careers in the market place to provide business, industry, and government with the mathematical support for which they are finding increasing need.
—MINA REES

433. Mathematical examination problems are usually considered unfair if insoluble or improperly described, whereas the mathematical problems of real life are almost invariably insoluble and badly stated. In real life, the mathematician's main task is to formulate problems by building an abstract model consisting of equations which shall be simple enough to solve without being so crude that they fail to mirror reality. Solving equations is a minor technical matter compared with this fascinating and sophisticated craft of model building and calls for both clear keen commonsense and highest quality of artistic and creative imagination. —J. HAMMERSELAY

434. The growing complexity of modern industry increases the opportunity for mathematicians to fit themselves in and become most valuable to industry. The maximum amount of thought must be given to design before it is built or to an experiment before it is started and this often involves considerable mathematical study. —W.E. SCOTT

435. Mathematics is an entity. The welfare of each part is essential to the other. Wisdom and farsightedness demand that all mathematicians join forces in attracting more students to the study of mathematics, and in offering them a broad view of the role of mathematics in our civilization—in research, in teaching and in industrial applications. We should recognize industrial mathematicians for what they are, a source of power for mathematics, a reason for the respect in which pure mathematics is held by many who do not understand it, and sometimes a leavening influence in determining the directions of interesting new mathematical research.

—MINA REES

436. The most vitally characteristic fact about mathematics is its quite peculiar relationship to the natural sciences, or more generally to any science which interprets experience on a level higher than purely descriptive level. —VON-NEUMANN

437. To isolate pure mathematics from the applied is like isolating the giant Antaeus from his mother Terra, the earth, contact with which gave him renewed strength from every fall in all his contests. To leave pure mathematics to the high heaven of passive contemplation, away from the applications, is to stifle it, very much as Herculeus stifled Antaeus in the mid-air. Therefore, instead of isolating the pure and applied aspects of mathematics we must endeavour to understand their relation and thereby the relation of mathematics to reality. —JAGJIT SINGH

438. We must not treat mathematics as a game of manipulating arbitrary symbols, but as a product of human societies whose members cooperate socially to advance civilization and culture. This means that mathematics has sociological, psychological, pragmatic and empirical aspects, besides the syntactical aspect with which alone the positivists seem to be concerned. —JAGJIT SINGH

439. We must remember that while mathematics has to resort to abstractions to secure a foothold for a first peep into reality, it must continually transcend them to get a fuller view of reality. —JAGJIT SINGH

440. If mathematics is an arbitrary construct of symbols, how could it ever be applied and used to predict events ? It is no answer, as Kaltsoff has remarked, to say that experience might have verified the prediction, for the point is that it did.
—JAGJIT SINGH

441. The narrow isolated specialist in mathematics does without doubt make a contribution, but he is not to be compared with the great mathematicians of history who have always understood and used the empirical sources of mathematical inspiration. Not every application is of genuine mathematical interest, but this does not justify the pure mathematician's tendency to make a virtue of a lack of interest in mathematical problems arising in other sciences. —J. P. LA SALLE

442. It seems to me that a worthwhile distinction can be drawn between two types of pure mathematics. The first, which unfortunately is somewhat out of style at present, centres attention on particular functions and theorems. The second is concerned with form and structure. Mathematics of the second kind hardly ever yields great and memorable results like the prime number theorem and Euler's formula. On the contrary, its theorems are generally small parts of a much larger whole and derive their main significance from the place they occupy in that whole. In my opinion, if a body of mathematics like this is to justify itself, it must possess aesthetic qualities akin to those of a good piece of architecture. It should have a solid foundation, its walls and beams should be firmly and truly placed, each part should bear a meaningful relation to every other part and its towers and pinnacles should exert the mind.

—G. F. Simmons

443. The immense problem of planning and running a socialist economy cannot be fully solved unless the technical and mathematical basis of planning is seriously developed and consolidated... Soviet economists long held the mistaken notion that mathematics could not be used in planning a socialist economy because the problems involved in compiling and carrying out the plan were too complex and multifarious. This was to ignore the great importance of investigating the laws of socialist economics not simply in qualitative terms, but also in terms of quantitative inter-connections and ratios .. As an instrumental science, mathematics can in our own hand become an excellent and serviceable tool with which to help build communism. There is plenty of scope in a socialist economy for its use in calculating balance for the economy as a whole, in analysing relationship between different branches of the economy and in working out optimum plans for

the use of available resources. Problems of this kind can be solved only mathematically.
—N. S. Nemechone

444. Whether man's travels carry him into space or into theoretical science, his passport must be stamped with the mathematician's seal of approval. We find that today mathematics is both one of the exciting frontiers to be explored, and the road to success in scientific and technological developments. Physics, chemistry, engineering, business, psychology, medicine, genetics and dozen of other fields are conducting an increasing percentage of their work in this strange and potent language. —J. G. Kemeny

445. It is a basic principle in the study of mathematics, and one too seldom emphasized that a proof is not really understood until the stage is reached at which one can grasp it as a whole and see it as a single idea. In achieving this end, much more is necessary than merely following the individual steps in the reasoning. This is only the beginning. A proof should be chewed, swallowed, and digested, and this process of assimilation should not be abandoned until it yields a full comprehension of the overall pattern of thought. —G. F. Simmons

446. Experience and intuition, though usually obtained more painfully, may be developed by mathematical insight. —R. Aris

447. Mathematical analysis consists in first breaking down the situation into its simplest component parts and then finding a mathematical description both for these and for their interactions. Experienced judgment is then needed to simplify the equations to a point where they are amenable to mathematical analysis without destroying some essential feature in the process. —R. Aris

448. It frequently happens that to get a set of manageable equations, the final model is idealised to a much greater degree than one would wish. However,

provided the dominant features are present, the resulting analysis is not valueless for it is very often true that a qualitative understanding of the real problem can be obtained from the quantitative solution of the idealised one. —R. Aris

449. The science of physics does not only give us (mathematicians) an opportunity to solve problems, but also helps us to discover the means of solving them, and it does this in two ways ; it leads us to anticipate the solution and suggests suitable lines of argument. —Henri Poincare

450. Just as a mountaineer climbs a mountain—because it is there, so a good mathematics student studies new material—because it is there.

—James B. Bristol

451. One cannot escape the feeling that mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was orginally put into them. —Hemrich Hertz

452. It therefore seems reasonable to expect that progress in the social sciences will have to be matched by still further strides in the expansion of the realm of mathematics. I fully expect that within the next century, the social sciences will serve as the major source of new problems to stimulate the mathematicians' creative imagination. —J. G. Kemeny

453. But I believe that the greatest opportunity for a significant contribution at the moment lies in the expert mathematician who is willing to devote his life to the exploration of a social science.

—J. G. Kemeny

454. The biological and social sciences are beginning to vie with physical sciences for the service of mathematician. Many papers in medicine, psychology and economics are unreadable for the mathematically uninitiated. —J. G. Kemeny

455. Many mathematicians have impression that mathematical problems in the social sciences are entirely trivial. On the contrary, most problems in social sciences are too difficult for present day mathematics. It is because that problems arising in the social sciences rapidly become difficult—that only some of the very simplest mathematical problems have been solved so far. Some day the theoretical social scientists will have to know more mathematics than the physicists need to know today.

—J. G. KEMENY

456. Often social scientists may be in agreement in requirements for the solution of a certain problem, even when no agreement can be reached in an actual solution. In such cases a mathematician must be consulted. He may show them that it is impossible to meet all the requirements they have laid down, and in that case they would have to agree on ways for asking for less or very likely he may tell them that there are infinitely many ways of solving these problems and give them some indication of the kind of additional requirement they would make on a solution.

—J. G. KEMENY

457. The bringing together of theory and practice leads to the most favourable results ; not only does practice benefit, but sciences themselves develop under the influence of practice, which reveals new subjects for investigation and new aspects of familiar subjects.

—P. L. CHEBYOHEV

458. Although the harassed engineers and economists would certainly like to see a cook book of mathematical recipes for all purposes, the mathematician, in his secret heart would like to see no such thing. This last named individual is for ever delighted to see the close of one investigation afford the entry to a dozen others, the scaling of one height leading merely to the discovery of further mountain ranges. Fortunately, the physical universe very kindly provides him with a hierarchy of problems leading

to new theories, and these new theories to further problems.

—R. Bellman

459. The construction of an adequate mathematical theory of control processes thus represents a resounding challenge to the mathematician. As always in the study of fundamental scientific problems, he will be amply rewarded by the many novel, interesting and significant mathematical problems which he encounters. In return for letting him play his game on his own terms, society will be rewarded by new scientific techniques, and new aesthetic and cultural experiences. Only this constant interchange between the real and abstract—a feedback control process in itself—keeps mathematics vital. Without it there are the dangers of sterility, atrophy and ultimately decadence.

—R. Bellman

460. So also the games in themselves merit to be studied and if some penetrating mathematician meditated upon them, he would find many important results, for man has never shown more ingenuity than on his plays.

—Leibnitz

461. To the mathematician, we would like to present a vista of vast new regions requiring exploration, where significant problems can be simply stated but require new ideas for their solution. In almost every direction taken, practically every thing remains to be done. There are concepts to make precise, arguments to make rigorous, equations to solve and computational algorithms to devise.

—R. Bellman

462. A mathematical model of a physical process to be used as long as it yields prediction in reasonable accordance with observation. Considering the complexity of actual physical phenomena *vis-a-vis* the simplicity of our mathematical modes, it is not surprising that we are forced to modify our formulations from time to time in order to obtain more accurate results. What is remarkable is that understanding of many physical processes that can be obtained from rudimentary assumptions.

—R. Bellman

463. The beauty of mathematics lies in the fact that it furnishes a universal language which characterises in a uniform way processes of quite different physical appearances. To some extent it transcends the narrow specialisation which regretably and irresistably mar our culture and to a greater extent, it, together with music, furnishes a bond between people over whose cradles lullabies in many different languages were sung. —R. BELLMAN

464. Who has taught us the true analogies, the profound analogies which the eyes do not see, but which reason can divine ? It is the mathematical mind, which scorns content and clings to pure form.
—HENRI POINCARE

465. The mathematician is perfectly willing to adopt whichever pose yields mastery over the process. This is the great advantage of the abstract formulation, the 'as if' attitude that enables the mathematician to focus the powerful techniques of a dozen different mathematical and physical theories upon one particular process. —R. BELLMAN

466. This is, after all, the essence of the abstract method. Properties of symbols which at first are consequences of the origin of the symbols become eventually properties of the abstract symbols, and then hold whenever the symbols occur. It is this which makes it profitable to consider as many different physical settings of the same mathematical equations as possible. Each different stage casts a different light upon the nature of the solution, each illuminates a different phase of its character. Since the total effect is cumulative, the greater the contacts with different aspects of reality, the greater the understanding of both the abstract and the applied.
—R. BELLMAN

467. We have discovered in the past decade that thinking and decision, are not solely the province of the mataphysicist, but are appropriate subjects for scientific enquiry. They have been rendered so, to a considerable extent, by the very authors of this volume. It is no coincidence that of these twelve

authors, nine are professional mathematicians, and the other three are well known as competent mathematicians. For it is primarily by means of mathematical techniques that these subjects have been brought to the stage where objective, quantitative, scientific methodology can be applied to them. Out of that methodology will inexorably come the applications and the understandings which are the twin goals sought.

—R. E. MACHOL

468. Recent years have witnessed an impressive growth of results, prestige and future expectation associated with what has been termed as 'quantitative', 'objective' or 'scientific' approach to problems of the real world. The boom is descended from the contacts between the mathematician's world and the real world, established during world war II on a scale hitherto unprecedented and since propagated in all directions.

—G. W. BROWN

469. A result of applied mathematics is only as reliable as the assumptions from which it is derived.

—C. E. SHANNON

470. A mathematician knows how to solve a problem—he cannot solve it.

—W. E. MILNE

471. Mathematical concepts turn up in entirely unexpected connections. They very often permit an unexpectedly close and accurate description of the phenomena in these connections. Also since we do not understand the reason of their usefulness, we cannot know whether a theory formulated in terms of mathematical concepts is uniquely appropriate. We are in a position similar to that of a man who was provided with a bunch of keys and who, having to open several doors in succession always hit on the right key on the first or second trial. He became sceptical concerning the uniqueness of the coordination between keys and doors.

—E. P. WIGNER

472. Mathematics is the science of skilful operations with concepts and rules invented just for this purpose. The principal emphasis is on the invention of concepts. Mathematics would soon run out

of interesting theorems if these had to be formulated in term of the concepts which already appear in the axioms. Much more advanced mathematical concepts have to be devised with a view of permitting ingenious logical operations which appeal to our aesthetic sense both as operations and also on their results of great generality and simplicity. These concepts are not chosen for their conceptual simplicity, but for their amenability to clever manipulations and to striking brilliant arguments.
—E. P. WIGNER

473. The role of evaluating the consequences of already established theories is not the most important role of mathematics in physics. Mathematics, is not so much the master of the situation in this function, it is merely serving as a tool. A more sovereign role for mathematics is implied in the statement that laws of nature must already be formulated in the language of mathematics to be an object for the use of applied mathematics. —E. P. WIGNER

474. This shows that the mathematical language has more to command it than being the only language which we can speak. It shows that it is, in a very real sense, the correct language.
—E. P. WIGNER

475. The miracle of the appropriateness of the language of the mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We hope that it will remain valid in future and that it will extend, for better or for worse, to our pleasure even though perhaps also to our bafflement, to wide branches of learning. —E. P. WIGNER

476. All these difficulties are but the consequences of our refusal to see that mathematics cannot be defined without acknowledging its most obvious feature, namely that it is interesting. —M. POLANYI

477. Great political and cultural surges have always reflected themselves in corresponding bursts of scientific and mathematical creativity. One aspect of

current intellectual explosion has been that the fields far removed from the conventional domains of mathematics and its applications are now being studied and cultivated with great zeal. Among these we may mention economics, management, organisation theory, psychology and biology and medicine.

—R. Bellman

478. It is felt that we have reached a point in almost every sphere of human activity and quasi-achievement where all major problems of business, industry and government can be formulated and presented as systems of mathematical logic.

—F. G. Smith

479. The cosmos of the poet, the painter and the writer will continue to be expressed in a qualitative manner, while the natural scientists and the system analyst require for the expression of their cosmos a quantitative form that only mathematical language can bring. Work in the field of mental abstraction demands the highest possible standard of rigorous precision in the description of relations.

—D. N. Chorafas

480. In the broadest sense, mathematics may be separated into two classes. One class deals with the symbols, their combinations and properties in a formal way. The other class concerns itself with the meaning of the symbols and the significance of the systems related to the real world.

—D. N. Chorafas

481. The aim of theoretical science is neither action nor production, but the acquiring of scientific truth. Of the three theoretical sciences—philosophy, mathematics and physics, mathematics made the earliest advance with respect to accuracy and truth.

—D. N. Chorafas

482. Never before has the scientific man been confronted with so many problems at once, never before have the frontiers of his knowledge been pushed so far that the thirst of the existing powerful minds would be insufficient to carry them far in frontal attack.

Never before has the urgency for the 'new push' been so great. That is exactly where mathematical simulation comes in. We use it as a tool in experimentation, as an extension of the scientist's penetrating power in problems of complexity. We use it as a means for testing solutions to find the 'truth'. We use it as the electronic microscope of technological and other studies. Most important of all, we use it as a lever in compressing time. For today, time is a premium. —D. N. CHORAFAS

483. When mathematics is applied to the solution of some problems, it is necessary to describe mathematically the system to be studied. Such description is often referred to as a mathematical model. Because it is never possible to represent the real world with complete accuracy, certain approximations and simplifications must be made when constructing a mathematical model. There are many reasons for this. One is that it is essentially impossible to find out what the real world is really like. Another is that a very accurate model of the real world can become impossibly difficult to work with mathematically. A final reason is that accurate models often cannot be justified on economic grounds. Simple approximate ones will yield results which are good enough so that the additional improvement obtained from a better model is not sufficient to justify the additional cost. —G. HADLEY & T. M. WHITIN

484. Mathematics is truly a splendid science, but mathematicians often are not worth a nickel ; very often the so-called mathematician expects to be considered a great thinker ; yet the greatest blockheads are among them, unfit for any kind of occupation which requires contemplation if it cannot be done directly though easy combination of symbols which is more the work of routine than of thought. —LICHTENBERG

485. Sharp distinctions are made between the concepts of mathematics which are ideas that we think about and the symbols of mathematics, which are marks

that we write in order to communicate with one another. Doing mathematics consists of thinking about concepts rather than arranging symbols upon paper. —C. F. Brumfield, R. F. Eichitz & M. E. Shanks

486. God exists since mathematics is consistent and the Devil exists since we cannot prove it.
—Andre Weil

487. Doing mathematics is a human activity, an important and necessary one, but human and not conversation with supernatural beings.
—F. W. Levi

488. Mathematics today is an 'esoteric' science which is unintelligible to many researchers in other departments, often even to those whose particular field is closely related to mathematics. The case of mathematician is perhaps similar to that of an expert in some little-known oriental language.
—Meschkowski

489. The investigation of a problem consists of scraping away all unwanted information until only the essential facts remain. The less you are told, the easier it is to find a solution. In elementary mathematics, we have a hotch potch of details. In higher mathematics we isolate the elements involved and study each by itself. Higher mathematics can be much simper than elementary mathematics.
—W. W. Sawyer

490. To pursue mathematical analysis while at the same time turning one's back on its applications and on intuition is to condemn to it hopeless atrophy.
—R. Courant

491. A large part of the history of physical science is a record of continuous human striving for a formulation of concepts which will permit description of the real world in mathematical terms. The more recent history of social sciences also reveals a determined attempt to arrive at more quantitatively substantiated theories through the use of mathematics.
—G. Hadley

492. Mathematics epitomizes the scientific genius which has created western civilization, it is not and it should not be regarded by its guardians as merely a servant of the technologies.

—Budden & Wormell

493. Mathematical idealism is therefore essentially a belief in the value of a certain quality of thinking directed towards the solution of a range of inherently difficult problems. Mathematics is a way of looking at the world accurately, thoroughly, economically and with the greatest possible recognition of generality. —Budden & Wormell

494. To present mathematics as a narrow utilitarian discipline is in effect, to substitute a financial carrot for the mathematical ideal. Mathematics is a very long and strange path to financial reward, while the ideal of mathematics has inspired two and a half millennia of mathematical progress.

—Budden & Wormell

495. It is difficult to imagine how by appeal to observations many of the postulates of current mathematics could either be verified or shown to be unsuitable, and one can only conclude that much of modern mathematics is not related to science but rather appears to be more closely related to the famous scholastic arguing of the middle ages.

—R. W. Hamming

496. Mathematicians (quite properly) attach much importance to elegance, deep results, important theorems and so forth. However mathematicians tend to identify elegance with surprise, and hence to arrange their final presentation in a surprising manner rather than in a manner which would tend to reveal how it was found. As a result the poor student enjoys the elegant presentation without finding out how to do mathematics. —R. W. Hamming

497. Mathematicians tend to put great emphasis on existence theorem whose purpose is to show that what they are talking about actually exists. Unfortunately, all too often the method of proof is non-